VanWyk's 103

Section 1.1 Homework Problems

TERMS YOU SHOULD KNOW: propositions, truth values, negations, conjunctions, disjunctions, DeMorgan's Laws, logical equivalence.

- 1. Which of the following are propositions? What is the truth value for each that is a proposition?
 - (a) Pizza is good.
 - (b) Today is Tuesday.
 - (c) Pizza is good or today is Tuesday.
 - (d) For each number x, there is some number y such that x + y = 0.
- 2. Negate each of the following compound propositions.
 - (a) $1+1=3 \text{ or } x \ge 5.$
 - (b) 1 + 1 = 3 and $x \ge 5$.
- 3. Assume both p and r are true propositions and that q is a false proposition. Find the truth value of each of the following propositions:
 - (a) $(\sim p) \lor q$.
 - (b) $p \land (q \lor (\sim r)).$
 - (c) $x \lor (\sim x)$, where x is any of p, q, or r.
 - (d) $x \wedge (\sim x)$, where x is any of p, q, or r.
- 4. Let p, q, and r be the following propositions:
 - p: You have the mumps.
 q: You miss the final.
 r: You pass the course.

Write " $(p \land q) \lor ((\sim q) \land r)$ " as an English sentence.

- 5. Let p and q be propositions. Use truth tables to show that $\sim (p \lor (\sim q))$ is logically equivalent to $(\sim p) \land q$.
- 6. Let p, q, and r be propositions. Use DeMorgan's Laws a bunch of times to rewrite $\sim [(p \land q) \lor ((\sim q) \land r)]$ so that the only negated expressions are (possibly) p, q, and r.

1a. This is not a proposition; it's an opinion.

1b. This is a proposition that is true if and only if you read it on Tuesday.

1c. This is not a proposition since the first part isn't.

1d. This is a true proposition; y = -x works every time.

2a. $1 + 1 \neq 3$ and x < 5. 2b. $1 + 1 \neq 3$ or x < 5.

3a. $F \lor F$, which is F. 3b. $T \land (F \lor F)$, which is F. 3c. $T \lor F$ or $F \lor T$, which is always T. 3d. $T \land F$ or $F \land T$, which is always F.

4. You have the mumps and you miss the final, or you make it to the final and you pass the course.

5.

p	q	$\sim p$	$\sim q$	$p \lor (\sim q)$	$\sim (p \lor (\sim q))$	$(\sim p) \land q$
Т	Т	F	F	Т	F	F
Т	F	\mathbf{F}	Т	Т	\mathbf{F}	\mathbf{F}
F	Т	Т	F	\mathbf{F}	Т	Т
\mathbf{F}	F	Т	Т	Т	\mathbf{F}	F

6.
$$((\sim p) \lor (\sim q)) \land (q \lor (\sim r))$$