VanWyk's 103 **Section 1.2 Homework Problems**

TERMS YOU SHOULD KNOW: *universal quantifier, existential quantifier*

1. Assume the "universe of discourse" is the set of integers $\{\ldots,-2,-1,0,1,2,\ldots\}$. True or false:

   (a) For some x, $x+1>x$.

   (b) For all x, $x+1>x$.

   (c) For some x, $x<2$.

   (d) For all x, $x<2$.

   (e) For each number x, there exists a number y such that $x+y=0$.

   (f) There exists a number x such that for each number y, $x+y=0$.

2. Rewrite each of the following sentences using universal quantifiers ("all," "each," "every") and/or existential quantifiers ("there exists," "some," "at least one").

   (a) A student in this class speaks German.

   (b) Students need to take 103.

   (c) Cows have four legs.

   (d) A student in this class has taken every course in one of the departments at JMU.

3. Negate each of the following propositions.

   (a) For all x, $x+1>x$.

   (b) For some x, $x<2$.

   (c) For each number x, there exists a number y such that $x+y=0$.

   (d) There exists a number x such that for each number y, $x+y=0$.

   (e) For each number x, there exists a number y such that $x+y=0$ or $xy=1$.

   (f) There exists a number x such that for each number y, $x+y=0$ and $xy=1$.

(g) All monkeys are curious, but no monkey is as curious as George.

4. Do the following pairs of propositions have the same logical meaning? If not, why not?

    (a) i. For each number x, there exists a number y such that $x+y=0$.

        ii. There exists a number x such that for each number y, $x+y=0$.

    (b) i. For every time, there is a season.

        ii. There is a season for every time.

    (c) i. For every number x, for every number y, $x+y=y+x$.

        ii. For every number y, for every number x, $x+y=y+x$.

1a. True.
1b. True.
1c. True; $x = 1$ works, for example.
1d. False; $x = 5$ is a counterexample.
1e. True; $y = -x$ works.
1f. False. No matter what x is, you can always find a number y so that $x + y \neq 0$. For example, $y = x + 1$ $(x + (x + 1) \neq 0)$.

2a. There exists a student in this class who speaks German.
2b. All students need to take 103.
2c. All cows have four legs.
2d. There exists a student in this class and there exists a department at JMU such that the student has taken every course in that department.

3a. For some x, $x + 1 \leq x$.
3b. For all x, $x \geq 2$.
3c. There exists an x such that for all y, $x + y \neq 0$.
3d. For all x, there exists a number y such that $x + y \neq 0$.
3e. There exists a number x such that for each number y, $x + y \neq 0$ and $xy \neq 1$.
3f. For each number x, there exists a number y such that $x + y \neq 0$ or $xy \neq 1$.
3g Some monkey's aren't curious, or some monkeys are as curious as George.

4a. No, these do not have the same meaning. In the second proposition, there is a *single* number y that "works" for all xs; in the first proposition, there may be as many ys as there are xs. (Also notice that the first proposition is true while the second is false.)
4b. No, these do not have the same meaning for the same reason as 4a: in the second statement, a single season "works" for all possible times.
4c. Yes, these have the same meaning. Universal quantifiers "commute" with each other.