VanWyk's 103 **Section 1.3 Homework Problems**

TERMS YOU SHOULD KNOW: *implication, converse, contrapositive, tautology, contradiction.*

1. State the hypothesis, conclusion, converse, and contrapositive of the following implications.

   (a) If today is Tuesday, then $2 + 1 = 5$.

   (b) If n isn't a perfect square, then $\sqrt{n}$ is irrational.

   (c) The sea is boiling hot if pigs have wings.

   (d) All men are pigs. (*Note: write it as an implication first.*)

   (e) If $x^2 = 100$, then $x = 10$ or $x = -10$.

   (f) If M is generated by X, then every a in M can be written as a word in X.

2. Use truth tables to determine whether the given compound proposition is a tautology, a contradiction, or neither.

   (a) $p \Rightarrow p$

   (b) $(p \wedge q) \wedge (\sim p)$

   (c) $\sim (p \vee q) \Leftrightarrow ((\sim p) \wedge (\sim q))$

   (d) $p \Rightarrow (q \Rightarrow r)$

3. Which of the following propositions are logically equivalent?

   (a) If John is 20 years old, then Mary is 21 years old.

   (b) If John isn't 20 years old, then Mary isn't 21 years old.

   (c) If Mary is 21 years old, then John is 20 years old.

   (d) If Mary isn't 21 years old, then John isn't 20 years old.

4. Negate the following implications:

   (a) If some dogs are reptiles, then all men are pigs.

   (b) If $x \leq 25$, then $x \leq 5$ and $x \geq -5$.

1a. **Hypothesis:** Today is Tuesday. **Conclusion:** $2+1=5$. **Converse:** If $2+1=5$, then today is Tuesday. **Contrapositive:** If $2+1 \neq 5$, then today isn't Tuesday.

1b. **Hypothesis:** n isn't a perfect square. **Conclusion:** $\sqrt{n}$ is irrational. **Converse:** If $\sqrt{n}$ is irrational, then n isn't a perfect square. **Contrapositive:** If $\sqrt{n}$ is rational, then n is a perfect square.

1c. **Hypothesis:** Pigs have wings. **Conclusion:** The sea is boiling hot. **Converse:** If the sea is boiling hot, then pigs have wings. **Contrapositive:** If the sea isn't boiling hot, then pigs don't have wings.

1d. This is equivalent to *If something is a man, then something is a pig.* **Hypothesis:** Something is a man. **Conclusion:** Something is a pig. **Converse:** If something is a pig, then something is a man. **Contrapositive:** If something isn't a pig, then something isn't a man.

1e. **Hypothesis:** $x^2 = 100$ **Conclusion:** $x = 10$ or $x = -10$. **Converse:** If $x = 10$ or $x = -10$, then $x^2 = 100$. **Contrapositive:** If $x \neq 10$ and $x \neq -10$, then $x^2 \neq 100$.

1f. **Hypothesis:** M is generated by X. **Conclusion:** Every a in M can be written as a word in X. **Converse:** If every a in M can be written as a word in X, then M is generated by X. **Contrapositive:** If some a in M cannot be written as a word in X, then M is not generated by X.

2a. This is a tautology. Notice all truth values of $p \Rightarrow p$ are T's.

p	$p \Rightarrow p$
T	T
F	T

2b. This is a contradiction. Notice all the truth values of $(p \wedge q) \wedge (\sim p)$ are F's.

p	q	$\sim p$	$p \wedge q$	$(p \wedge q) \wedge (\sim p)$
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

2c. This is a tautology. Notice the last two columns have identical truth values.

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim (p \vee q)$	$(\sim p) \wedge (\sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

2d. This is neither a tautology nor a contradiction, since its truth value depends on the truth values of p, q, and r (there is a lone F in the second spot). Notice we need eight columns to enumerate all the possibilities because there are three different constituent propositions (p, q, and r).

p	q	r	$q \Rightarrow r$	$p \Rightarrow (q \Rightarrow r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

3. 3a and 3d are logically equivalent, as are 3b and 3c. An implication is always logically equivalent to its contrapositive.

4a. Some dogs are reptiles and some men are not pigs.
4b. $x \leq 25$ and ($x > 5$ or $x < -5$).