TERMS YOU SHOULD KNOW: *sets, intersection, union, complement, Cartesian product.*

1. Let $\mathcal{U} = \{1, 2, 3, \ldots, 9, 10\}$ be the universe of discourse, let $A = \{2, 4, 6, 8\}$, and let $B = \{1, 2, 3\}$. Find each of the following sets.

   (a) $A \cup B$.

   (b) $A \cap B$.

   (c) A'.

   (d) B'.

   (e) $A \times B$.

   (f) $B \times A$.

2. Assume $A, B \subset \mathcal{U}$ (the universe of discourse), $|A| = 25$, $|B| = 10$, and $|\mathcal{U}| = 100$. (Recall $|X|$ denotes the number of elements in the set X.)

   (a) What is $|A'|$? $|B'|$?

   (b) What is $|A \times B|$?

   (c) What is the smallest $|A \cup B|$ can be? What is the largest $|A \cup B|$ can be?

   (d) What is the smallest $|A \cap B|$ can be? What is the largest $|A \cap B|$ can be?

   (e) Suppose you also know that $|A \cap B| = 5$. What must $|A \cup B|$ be?

3. Let A, B, and C be nonempty sets contained in the same universal set $\mathcal{U}$. For each of the following, draw a single Venn diagram illustrating the given relationships. Be sure to label A, B, and C in your diagrams.

   (a) $A \subseteq C$, $B \subseteq C$, and $A \cap B = \emptyset$.

   (b) $A \subseteq B$ and $C = B'$.

   (c) $A \subseteq B$, $A \cap C = \emptyset$, and $B \cap C \neq \emptyset$.

   (d) $A \cap B \subseteq C$ and $A \cup B \nsubseteq C$.

4. Let $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ denote the set of natural numbers.

(a) List the elements of the set $A = \{n \in \mathbb{N} \mid 5 \le n \le 11\}$.

(b) List (some of) the elements of the set $B = \{2n \mid n \in \mathbb{N}\}$.

(c) List (some of) the elements of the set $C = \{2n+1 \mid n \in \mathbb{N}\}$.

(d) Write the set $D = \{0, 3, 6, 9, \dots\}$ using set-builder notation.

1a. $A \cup B = \{1,2,3,4,6,8\}$.
1b. $A \cap B = \{2\}$.
1c. $A' = \{1,3,5,7,9,10\}$.
1d. $B' = \{4,5,6,7,8,9,10\}$.
1e. $A \times B = \{(2,1),(2,2),(2,3),(4,1),(4,2),(4,3),(6,1),(6,2),(6,3),(8,1),(8,2),(8,3)\}$.
1f. $B \times A = \{(1,2),(1,4),(1,6),(1,8),(2,2),(2,4),(2,6),(2,8),(3,2),(3,4),(3,6),(3,8)\}$.

2a. $|A'| = 100 - 25 = 75$. $|B'| = 100 - 10 = 90$.
2b. $|A \times B| = (25)(10) = 250$.
2c. The smallest $|A \cup B|$ can be is $25 = |A|$; this occurs if $B \subseteq A$. The largest
$|A \cup B|$ can be is $|A| + |B| = 35$; this occurs if $A \cap B = \emptyset$.
2d. The smallest $|A \cap B|$ can be is 0; this occurs if $A \cap B = \emptyset$. The largest
$|A \cap B|$ can be is $|B| = 10$; this occurs if $B \subseteq A$.
2e. $|A \cup B| = |A| + |B| - |A \cap B| = 25 + 10 - 5 = 30$.

3a.

3b.

3c.

3d.
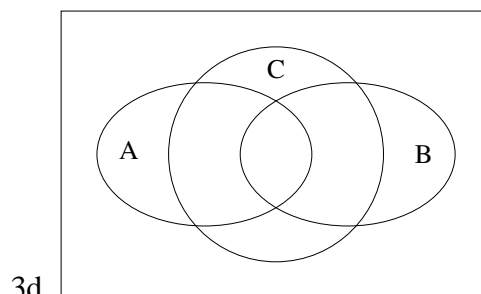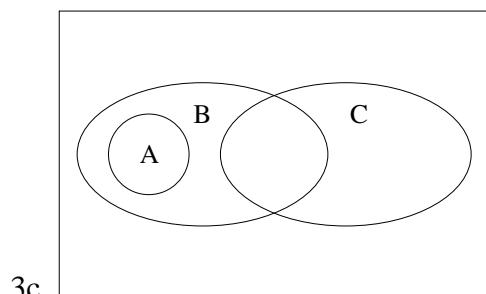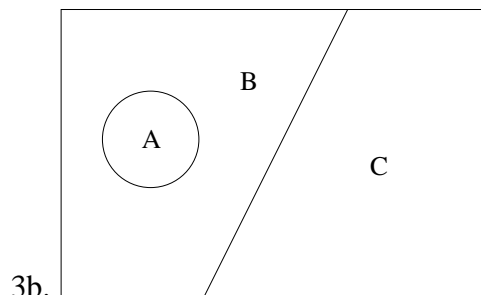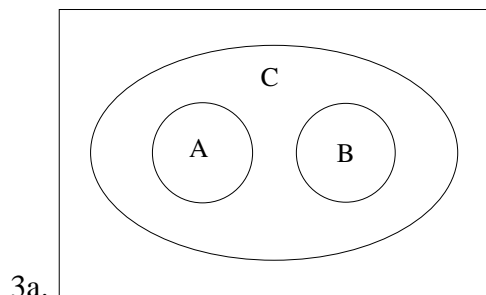
4a. $A = \{5,6,7,8,9,10,11\}$.
4b. $B = \{0,2,4,6,8,\ldots\}$.
4c. $C = \{1,3,5,7,9,\ldots\}$.
4d. $D = \{3n \mid n \in \mathbb{N}\}$.