VanWyk's 103 **Section 3.2 Homework Problems**

TERMS YOU SHOULD KNOW: *fundamental counting principle, permutations, combinations*

1. Let $\mathcal{A} = \{a, b, c, \ldots, x, y, z\}$ be the set of the 26 letters of the alphabet.
   (a) How many 5-letter "words" (whether or not they make sense) can be made from the elements of $\mathcal{A}$ if letters can be used more than once?
   (b) How many 5-letter "words" (whether or not they make sense) can be made from the elements of $\mathcal{A}$ if letters cannot be used more than once?
   (c) How many 5-element sets can be made from the elements of $\mathcal{A}$?
2. (a) How many possible 7-digit phone numbers are there if they can start with any number?
   (b) How many possible 7-digit phone numbers are there if they can start with any number except a 0 or a 1?
   (c) In the movies, all phone numbers start with 555. How many possible 7-digit phone numbers are there that start with 555?
   (d) What is the probability that a random phone number chosen from the set of all numbers that doesn't start with a 0 or a 1 starts with 555?
3. You draw 5 cards from a standard 52-card deck.
   (a) How many different hands are there?
   (b) If you draw only three cards, how many ways are there to draw 3 aces?
   (c) If you draw 5 cards, how many different ways are there to draw 3 aces?
   (d) What is the probability that a random 5-card hand is 3 aces?
4. There are 3 boys and 3 girls in line. How many different ways can they stand in order to have the order (front to back) boy, girl, boy, girl, boy, girl?
5. A bag contains 7 green balls and 5 yellow balls; all 12 are identical except for their color.
   (a) If you reach in and grab one ball at random, what is the probability it is yellow?
   (b) In how many ways can 2 balls be selected?

(c) In how many ways can 2 yellow balls be selected?

(d) If you reach in and grab two balls at random, what is the probability that both are yellow?

1a. 26^5
1b. ${}_{26}P_5$
1c. ${}_{26}C_5$

2a. 10^7
2b. $8 \cdot 10^6$
2c. 10^4
2d. $\frac{10^4}{8 \cdot 10^6} = \frac{1}{800}$

3a. ${}_{52}C_5 = 2,598,960$
3b. ${}_4C_3 = 4$
3c. This is a tough one. The number of ways to draw 3 aces is ${}_4C_3$. The number of ways to draw 2 non-aces is ${}_{48}C_2$. So the answer to the question is the product ${}_4C_3 \cdot {}_{48}C_2$.
3d. Divide the answer in 3c by the answer in 3a.

4. There are ${}_3P_3 = 6$ ways for the boys to be arranged and ${}_3P_3 = 6$ ways for the girls to be arranged, so the answer is $6 \cdot 6 = 36$.

5a. $\frac{5}{12}$
5b. ${}_{12}C_2 = 66$
5c. ${}_5C_2 = 10$
5d. $\frac{5}{33}$