VanWyk's 103 **Section 4.3 Homework Problems**

TERMS YOU SHOULD KNOW: *sense, direct, opposite, fixed-point*

1. Let H be a half-turn about point O and let G be a glide reflection whose axis of reflection contains O.

   (a) Is HG direct or opposite?

   (b) Does HG have any fixed points? If so, where?

2. Let H_1 and H_2 be half-turns about (different) points O_1 and O_2.

   (a) Is H_1H_2 direct or opposite?

   (b) Show H_1H_2 is a translation parallel to the line segment O_1O_2 and twice as long.

3. Let H be a half-turn and let T be a translation.

   (a) Is HT direct or opposite?

   (b) Does HT have any fixed points?

   (c) Use Problem 2b to show HT is a half-turn.

1a. Opposite.
1b. Yes, HG has fixed points.

Let's say the translational part of G moves points d units to the right. Then the entire vertical line located $\frac{1}{2}d$ units to the right of O is fixed. Draw a picture and convince yourself.

2a. Direct.
2b. Draw a picture (put O_1 and O_2 on a horizontal line) and follow two different points through the motion H_1H_2.

3a. Direct.
3b. Yes.
3c. Since T is a translation, it can be written as a product of two half-turns by Problem 2b. You can choose one of the half-turns to be about *any* point; once you do, the other half-turn's center of rotation is determined by T.

So, we can choose our first half-turn as H, and we can write $T = HH'$, where H' is the other half-turn as in the previous paragraph. But then $HT = H(HH') = H^2H' = H'$, so HT is a half-turn.