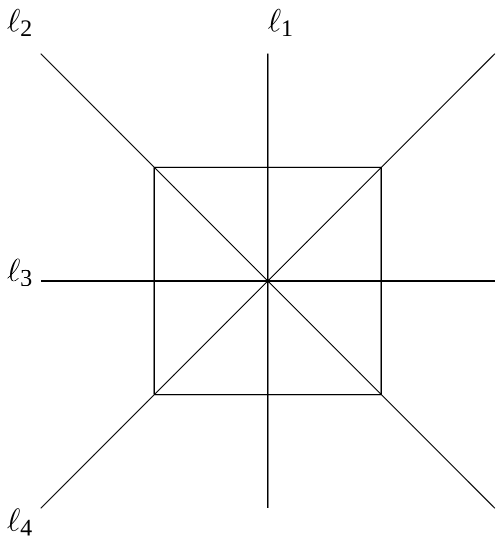
1. The symmetry group of the square is $D_4 = \{I, S, S^2, S^3, R_1, R_2, R_3, R_4\}$, where each reflection R_i is about the line ℓ_i below, and $S = R_1R_2$.



   (a) How many degrees does $S = R_1R_2$ rotate the square counterclockwise? How about R_1R_3? R_1R_4? R_2R_1?

   (b) Which elements of D_4 are direct? Opposite?

   (c) Make a "multiplication table" for the elements of D_4, i.e., fill in the following chart. You can use the fact (which we will see later) that every element of D_4 has to occur exactly once in each row and in each column. It might help to use a square card with numbers in each corner.

	I	S	S^2	S^3	R_1	R_2	R_3	R_4
I	I	S	S^2	S^3	R_1	R_2	R_3	R_4
S	S							
S^2	S^2							
S^3	S^3							
R_1	R_1				I			
R_2	R_2					I		
R_3	R_3						I	
R_4	R_4							I

2. In D_{231}, if you have the product $R_{45}S^{199}R_{19}R_{71}S^{22}$, is it some S^i or some R_i, and why?
3. In D_{180}, what is the angle between ℓ_1 and ℓ_2? What is the angle of minimal counterclockwise rotation (i.e., the angle of S)?

1a. S rotates the square 90° counterclockwise, since the angle between ℓ_1 and ℓ_2 is 45°. R_1R_3 rotates the square $2 \cdot 90^\circ = 180^\circ$ counterclockwise. R_1R_4 rotates the square $2 \cdot 135^\circ = 270^\circ$ counterclockwise. R_2R_1 rotates the square 90° *clockwise*, which is the same as 270° counterclockwise.
1b. The S^i's are always direct since they are rotations and the R_i's are always opposite, since they are reflections.
1c.

	I	S	S^2	S^3	R_1	R_2	R_3	R_4
I	I	S	S^2	S^3	R_1	R_2	R_3	R_4
S	S	S^2	S^3	I	R_4	R_1	R_2	R_3
S^2	S^2	S^3	I	S	R_3	R_4	R_1	R_2
S^3	S^3	I	S	S^2	R_2	R_3	R_4	R_1
R_1	R_1	R_2	R_3	R_4	I	S	S^2	S^3
R_2	R_2	R_3	R_4	R_1	S^3	I	S	S^2
R_3	R_3	R_4	R_1	R_2	S^2	S^3	I	S
R_4	R_4	R_1	R_2	R_3	S	S^2	S^3	I

Notice that this table illustrates how direct and opposite motions multiply:

	D	O
D	D	O
O	O	D

2. It's some R_i because the product contains an odd number of opposite motions (so it must be opposite).

3. The angle between ℓ_1 and ℓ_2 in D_{180} is $\frac{180^\circ}{180} = 1^\circ$. So, the angle of rotation of $S = R_1R_2$ is 2°.