1. Let $*$ be the binary operation defined on the set of natural numbers $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ given by
$$a * b = a^b.$$
Is $*$ commutative? Is $*$ associative?

2. Show the following binary operation $*$ on $S = \{0, 1\}$ is not associative:

$*$	0	1
0	1	0
1	1	1

Hint: 0^3.

3. Write the table for *left* projection on the set $S = \{a, b, c\}$.

4. Is the following a table for a commutative binary operation? Why or why not?

(a)

	a	b	c
a	c	b	a
b	b	c	a
c	c	a	c

(b)

	a	b	c
a	c	b	a
b	b	b	c
c	a	c	a

5. Give an argument showing the following is a table for an associative binary operation.

	a	b	c
a	b	b	b
b	b	b	b
c	b	b	b

6. There are 3^9 different binary operations on a set with 3 elements. How many *commutative* binary operations are there on a set with 3 elements?

   Hint: Use the symmetry of the table.

1. $*$ is not commutative since $2*3 = 2^3 = 8$, while $3*2 = 3^2 = 9$.

$*$ is not associative since $(2*3)*4 = (2^3)*4 = 8*4 = 8^4 = (2^3)^4 = 2^{12}$, while $2*(3*4) = 2*(3^4) = 2*81 = 2^{81}$.

2. $0*(0*0) = 0*1 = 0$, while $(0*0)*0 = 1*0 = 1$, so $*$ is not associative.

3. Since $\alpha * \beta = \alpha$, here is the table:

$*$	a	b	c
a	a	a	a
b	b	b	b
c	c	c	c

4a. No, since $a*c \neq c*a$.
4b. Yes, since the table is symmetric about the main diagonal.

5. Since $\alpha * \beta = b$, it follows that for all $\alpha, \beta, \gamma \in S$, $(\alpha*\beta)*\gamma = b = \alpha*(\beta*\gamma)$. So $*$ is associative.

6. The answer is 3^6. By the symmetry we need for commutativity, the table must look like

	a	b	c
a	x	u	v
b	u	y	w
c	v	w	z

So there are really only 6 slots we can fill. (Once the 6 on and above the main diagonal are filled in, the 3 below that are determined.)