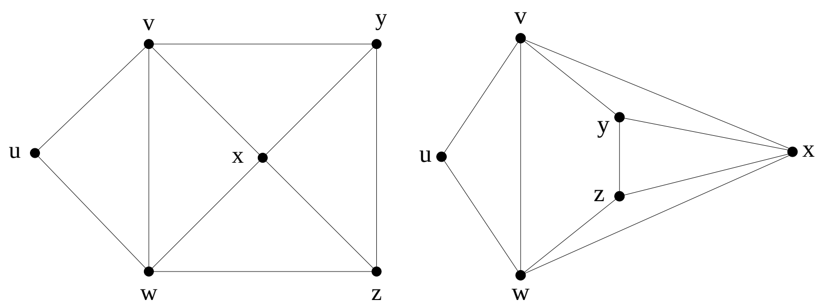
1. Here are two plane graphs, Γ_1 and Γ_2, respectively:



   (a) List the number of vertices, edges, and faces for both Γ_1 and Γ_2.

   (b) List the degrees of all the vertices of both Γ_1 and Γ_2.

   (c) List the degrees of all the faces (i.e., the number of edges incident with, or bounding, each face) of both Γ_1 and Γ_2.

   (d) Draw the duals Γ_1^* and Γ_2^*.

   (e) Repeat 1a–1c for Γ_1^* and Γ_2^*.

   (f) Conclude that Γ_1 and Γ_2 are "basically the same graph" but Γ_1^* and Γ_2^* are "structually different."

1a. Both have 6 vertices, 10 edges, and 6 faces.

1b. The degrees of the vertices for both Γ_1 and Γ_2 are the same:

vertex	u	v	w	x	y	z
degree	2	4	4	4	3	3

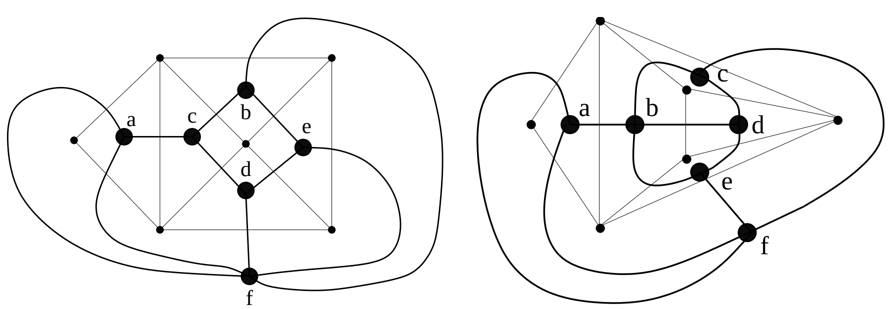
1c. The degrees of the faces for Γ_1 are:

face	uvw	vxw	vxy	wxz	xyz	$uvyzw$
degree	3	3	3	3	3	5

The degrees of the faces for Γ_2 are:

face	uvw	$vyzw$	vxy	xyz	wxz	$uvxw$
degree	3	4	3	3	3	4

1d. Here are Γ_1^* and Γ_2^*, superimposed over Γ_1 and Γ_2, respectively:



1e. Both have 6 vertices, 10 edges, and 6 faces. Notice, though, that the 6 vertices of the duals correspond to the 6 faces of the originals, and vice-versa.

For Γ_1^*:

vertex	a	b	c	d	e	f
degree	3	3	3	3	3	**5**
face	af	$acdf$	$bcde$	bef	def	$acbf$
degree	2	4	4	3	3	4

For Γ_2^*:

vertex	a	b	c	d	e	f
degree	3	4	3	3	3	4
face	af	$abef$	bde	bcd	$cdef$	$abcf$
degree	2	4	3	3	4	4

1f. It is easy to see that $\Gamma_1 \cong \Gamma_2$. Basically, Γ_2 is obtained from Γ_1 by grabbing the vertex x and pulling it out to the right.

The reason $\Gamma_1^* \ncong \Gamma_2^*$ is that Γ_1^* has a vertex of degree 5 but Γ_2^* doesn't. So there is *no way* Γ_1^* can be deformed into Γ_2^*, since the degrees of the vertices don't change under any deformation.

What this example shows is that two graphs that are "structurally the same" (*isomorphic*) can have duals that are not. So, it only makes sense to talk about duals of plane graphs and not in the more general context of planar graphs.