1. Draw the Cayley graph of the group $\mathbb{Z}_6$ with respect to the generating set $X = \{2, 3\}$.

2. Draw the Cayley graph of the group D_4 with respect to the generating set $X = \{R_1, R_2\}$. (Use the group table from Chapter 5.)

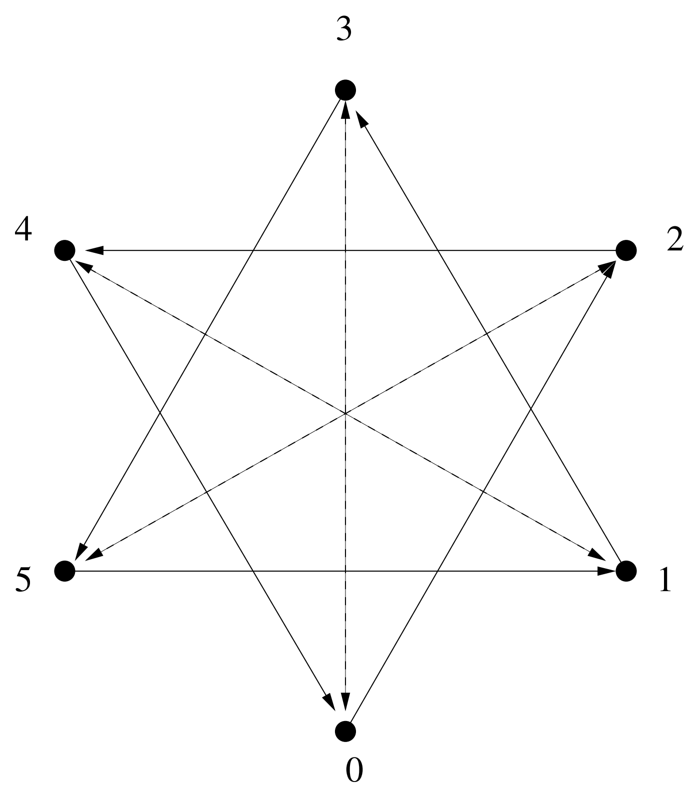
3. By assigning one of the twelve musical pitch classes to an element of $\mathbb{Z}_{12}$ in the "obvious" way (C $= 0$, C$^\sharp = 1$, etc.), we get this table of the "musical pitch class group", $\mathcal{M}$:

	C	C$^\sharp$	D	D$^\sharp$	E	F	F$^\sharp$	G	G$^\sharp$	A	A$^\sharp$	B
C	C	C$^\sharp$	D	D$^\sharp$	E	F	F$^\sharp$	G	G$^\sharp$	A	A$^\sharp$	B
C$^\sharp$	C$^\sharp$	D	D$^\sharp$	E	F	F$^\sharp$	G	G$^\sharp$	A	A$^\sharp$	B	C
D	D	D$^\sharp$	E	F	F$^\sharp$	G	G$^\sharp$	A	A$^\sharp$	B	C	C$^\sharp$
D$^\sharp$	D$^\sharp$	E	F	F$^\sharp$	G	G$^\sharp$	A	A$^\sharp$	B	C	C$^\sharp$	D
E	E	F	F$^\sharp$	G	G$^\sharp$	A	A$^\sharp$	B	C	C$^\sharp$	D	D$^\sharp$
F	F	F$^\sharp$	G	G$^\sharp$	A	A$^\sharp$	B	C	C$^\sharp$	D	D$^\sharp$	E
F$^\sharp$	F$^\sharp$	G	G$^\sharp$	A	A$^\sharp$	B	C	C$^\sharp$	D	D$^\sharp$	E	F
G	G	G$^\sharp$	A	A$^\sharp$	B	C	C$^\sharp$	D	D$^\sharp$	E	F	F$^\sharp$
G$^\sharp$	G$^\sharp$	A	A$^\sharp$	B	C	C$^\sharp$	D	D$^\sharp$	E	F	F$^\sharp$	G
A	A	A$^\sharp$	B	C	C$^\sharp$	D	D$^\sharp$	E	F	F$^\sharp$	G	G$^\sharp$
A$^\sharp$	A$^\sharp$	B	C	C$^\sharp$	D	D$^\sharp$	E	F	F$^\sharp$	G	G$^\sharp$	A
B	B	C	C$^\sharp$	D	D$^\sharp$	E	F	F$^\sharp$	G	G$^\sharp$	A	A$^\sharp$

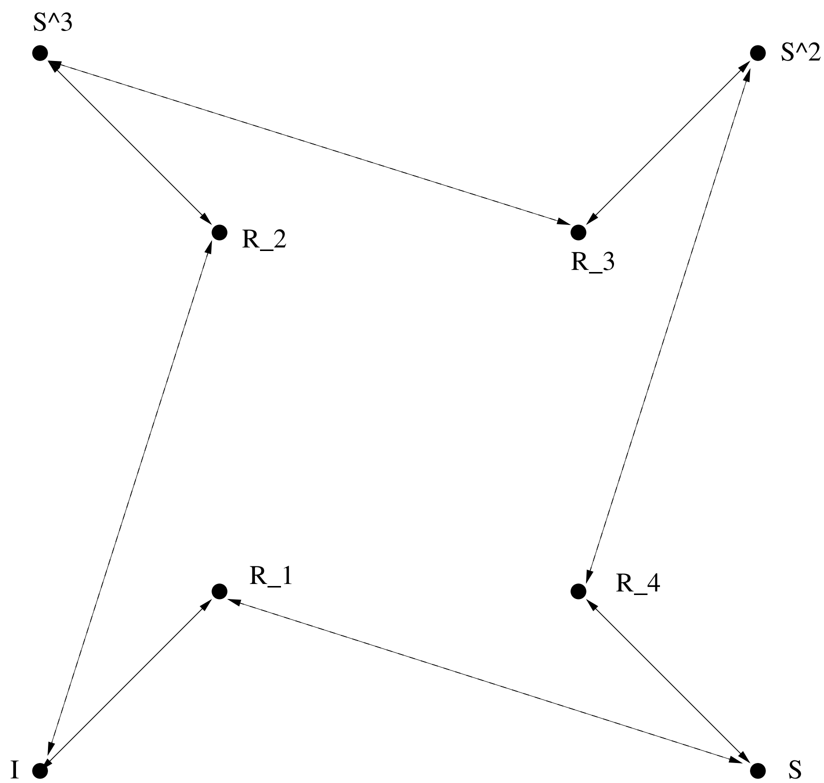
Put the 12 tones in a circle (in the usual way) and use the table above to draw the Cayley graph $\mathcal{C}(\mathcal{M}, \{F\})$. Then try it for $\mathcal{C}(\mathcal{M}, \{C\sharp\})$. Musicians should be able to interpret these graphs.

(Note: If you don't know or care about music, then just compute the Cayley graphs $\mathcal{C}(\mathbb{Z}_{12}, \{5\})$ and $\mathcal{C}(\mathbb{Z}_{12}, \{1\})$.)

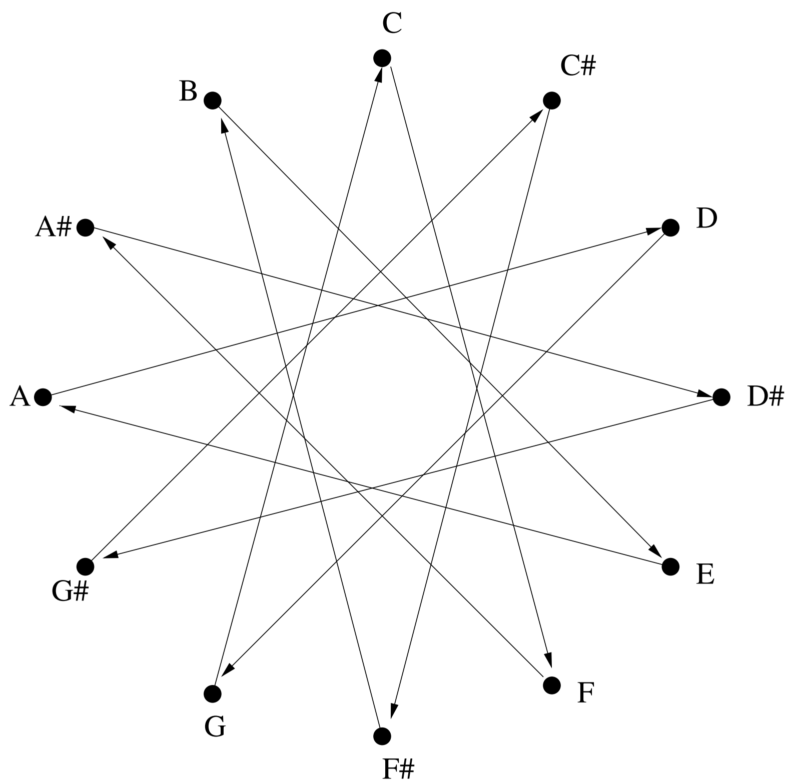
1. Here the solid lines correspond to the generator 2, while the dashed lines correspond to the generator 3. Notice, as usual, there is an edge of each type going in and coming out of each vertex.



2. Here the solid lines correspond to the generator R_1, while the dashed lines correspond to the generator R_2.

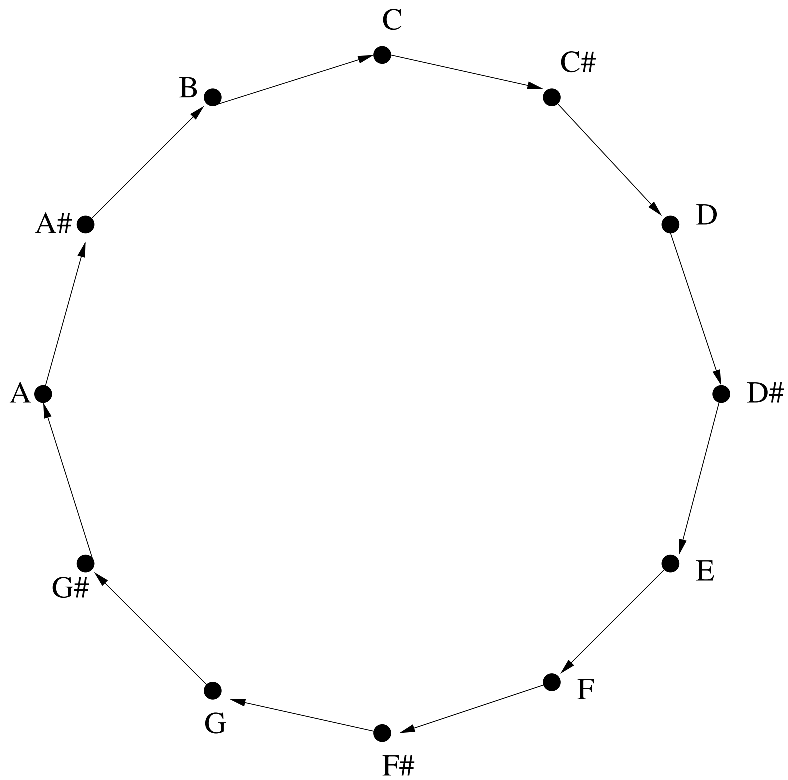


3. Here all the edges correspond to the generator F = 5.



This Cayley graph represents the “circle of fourths” (or the “circle of fifths” if you travel the edges backwards), i.e., by increasing by a perfect fourth each time. Musicians know that one way to practice something in all twelve keys is to travel around this circle, since it hits all the twelve tones.

In this graph, the edges correspond to the generator $C^\sharp = 1$.



This Cayley graph represents traveling around the twelve tones chromatically (i.e., by increasing by a half step each time), another way to practice something in all twelve keys.