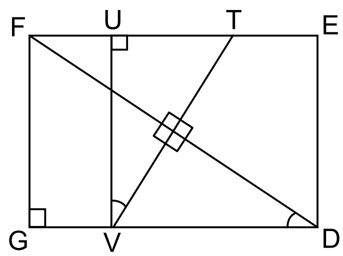
Solution to the 2003 puzzle by John R. H. Goering

To solve this problem, make use of the fact that the crease is the perpendicular bisector of the segment connecting the two opposite corners. With this in mind, a diagram like the following may be drawn:



In the drawing, $\overline{DF}$ is the diagonal, $\overline{VT}$ is the crease, and $\overline{UV}$ is perpendicular to the longer sides of the paper.

To work the problem, the following notation will help:

l = length of paper = DG
w = width of paper = $FG = UV$
c = length of crease = VT
d = length of diagonal = DF
r = ratio of length to width = l / w

Thus $l = rw$ (Equation 1)

A little bit of geometry shows that $\angle GDF \cong \angle UVT$ as is indicated in the diagram.
A little more geometry shows that $\Delta GDF \sim \Delta UVT$.
Thus $\frac{GD}{UV} = \frac{DF}{VT}$. Substituting the above notation gives $\frac{l}{w} = \frac{d}{c}$
Thus the ratio of the longer side to the shorter side is $r = \frac{d}{c}$ (Equation 2)
By the Pythagorean theorem, $d^2 = w^2 + l^2$. Substituting rw for l (see Equation 1) and then taking the square root of both sides gives $d = \sqrt{w^2 + r^2 w^2}$
Factoring out w^2 produces $d = w\sqrt{1+r^2}$ (Equation 3)
Since the crease is the same length as the longer side, $c = l$.
Substituting Equation 1 produces $c = rw$ (Equation 4)

Substituting Equations 3 and 4 into Equation 2 produces $r = \frac{w\sqrt{1+r^2}}{rw}$

Therefore $r = \frac{\sqrt{1+r^2}}{r}$. Thus $r^2 = \sqrt{1+r^2}$.
Squaring and then rearranging gives $r^4 - r^2 - 1 = 0$
Solving this quadratic in r^2 shows that $r^2 = \frac{1+\sqrt{5}}{2}$ which is the golden ratio!

Thus $r = \sqrt{\frac{1+\sqrt{5}}{2}}$ Q.E.D.