Abstracts of Past JMU REU Projects

- **Title.** *p-Coloring Classes of Torus Knots* (2004)

  Students. Anna-Lisa Breiland, Willamette College and Layla Oesper, Pomona College

  Mentor. Laura Taalman

  Abstract. We develop a theorem for determining the p-colorability of any (m,n) torus knot. We also prove that any p-colorable (m,n) torus knot has exactly one p-coloring class. Finally, we show that every p-coloring of the braid projection of an (m,n) torus knot must use all of the p colors.

- **Title.** *Analysis for Intracranial Saccular Aneurysms: I. Nonlinear anisotropic biomechanical model, II. Fast, adaptive, and accurate computational algorithm, and III. scientific visualization.* (2004)

  Students. James Collins, James Madison University and Matthew Watts, James Madison University

  Mentor. Paul Warne

  Abstract. Biomechanical modeling has great potential to contribute to and improve the requisite diagnostic and therapeutic capabilities of health care. Arterial wall mechanics is particularly crucial not only for the understanding of blood circulatory physiology but also for analyzing the mechanisms of vascular disorders. Nonlinear continuum mechanics is an excellent framework for formulating mathematical models that can accurately capture the complex behavior of biological soft tissues. Among other vascular disorders, intracranial saccular aneurysms have received attention as an open problem that merits increased research. A simple dynamic deformation studied by the continuum biomechanics community is one of uniaxial stretch in the radial direction. Using the approximations of membrane theory for example, Humphrey et al. study this problem assuming the wall of the sphere to be infinitesimally small. Instead, we study this problem for a finite, thin-walled sphere using the fully three-dimensional exact governing equations of nonlinear elasticity. The dynamics of our intracranial saccular aneurysm model are compared for three different material models: a classic neo-Hookean elastic strain energy, a simple Fung strain energy often used in modeling biological tissues, and a new strain energy capturing anisotropy of a radially fiber-reinforced material that suggests ways to control the growth of the aneurysm. Such modeling of physical and biological problems generally leads to complicated ODE-BVPs, unable to be solved analytically. Thus, accurate, efficient, and adaptable numerical methods are critical, especially since singularities and bifurcation phenomena are common. As accuracy can be significantly compromised near a singularity for many predominant nonlinear ODE numerical solution algorithms (e.g., Runge-Kutta based methods), here we also incorporate a tractable new algorithm (Algebraic-Maclaurin-Padé (AMP)) that approximates the solution to an ODE system by the Padé approximant at each step, and can produce much better accuracy in far less time near a singularity. We also develop adaptive time-stepping for the AMP method, using the error calculated from the Padé approximant, allowing for shorter calculation time while maintaining the same accuracy as with fixed time-stepping. Finally, we develop animations of the aneurysm dynamics from our computed numerical data.

- **Title.** *One Regularity of Cayley graphs* (2004)

  Students. Mary Balmes, St. Joseph's College and Jaclyn Kaminski, Xavier University

  Mentor. Jason Rosenhouse

  Abstract. The problem of finding highly symmetric graphs is almost as old as graph theory itself. We considered the problem of finding examples of one-regular Cayley graphs, meaning they are arc-transitive and each arc has a trivial arc-stabilizer. The first known cubic one-regular graph was found in 1952 by Frucht, and we examined his construction in detail. More recently Kwak and Oh found a method for constructing one- regular Cayley graphs of any even valency, and we considered their method as well. Examples of one-regular Cayley graphs of odd valency are harder to come by. We discovered several properties any such graph must satisfy. We also considered direct and semi-direct products of groups, and discovered certain properties such groups must have if they are to give rise to one-regular Cayley graphs.

- **Title.** *Using Scale Mixtures of Normals to Model Continuously Compounded Return* (2004)

  Students. Kristen Dardia, James Madison University and Melanie Wilson, Allegheny College

  Mentor. Hasan Hamdan

  Abstract. In this paper, a new method for es timating the parameters of scale mixtures of normals is presented. The new method is called UNMIX and is based on minimizing the weighted square distance between exact values of the density of the scale mixture and estimated values using kernel smoothing techniques over a prespecified grid of x-values and a grid of potential sigma values. Applications of the method are made in modeling the continuously compounded return of stock prices. Modeling this ratio with UNMIX proves promising in comparison with other existing techniques that use only one normal component, or those that use more than one component based on the EM algorithm as the method of estimation.

- **Title.** *Pretzel knots and colorability* (2003)

  Students. Kathryn Brownell, Lenoir-Rhyne College and Kaitlyn O'Neil, Merrimack College

  Mentor. Laura Taalman

  Abstract. We develop a formula for determining the number of fundamentally different ways that an m-colorable knot can be m-colored, based on the m-nullity of the knot. We then determine the m-nullity of any (p,q,r) pretzel knot, and thus a way to determine the m-colorability and number of fundamentally different m-colorings of any pretzel knot.

- **Title.** *k-alternating knots* (2003)

Students. Philip Hackney, Central Michigan University and Nathan Walters, Drake University

Mentor. Leonard Van Wyk

Abstract. A projection of a knot is *k-alternating* if its overcrossings and undercrossings alternate in groups of k as one reads around the projection (an obvious generalization of the notion of an alternating projection). More generally, a projection is *w-repeating* if, rather than alternating in groups of k, the crossings follow any pattern whatsoever. We show every knot that admits a k-alternating projection also admits a $(k+1)$-alternating projection, and that every knot that admits a w-repeating projection admits a k-alternating projection for $k \leq \frac{|w|}{2}$. We also prove the surprising result that every knot admits a 2-alternating projection, which partitions nontrivial knots into two classes: alternating and 2-alternating. Finally, we explore the notion of the *k-alternating excess* of a knot, the difference between the smallest number of crossings in a k-alternating projection of the knot and its crossing number.

- **Title.** *Padé approximates for torsion of a compressible nonlinearly elastic cylinder: modeling, computation, and visualization* (2003)

  Students. Danielle Miller, James Madison University and Jennifer Salyer, East Tennessee State University

  Mentors. Debra Warne and Paul Warne

  Abstract. The boundary-value problem (BVP) resulting from the equations of nonlinear elastostatics for torsion of a circular cylinder for a class of general Blatz-Ko materials involves a nonlinear, singular, 2^{nd} order ordinary differential equation for the radial deformation with non-standard boundary conditions. Using a new computational approach to project the BVP to a polynomial system, we are able to compute the MacLaurin coefficients of the solution to any degree and then accurately solve for the radial deformation by numerically creating its Padé approximation. Significant improvements in both accuracy and efficiency are shown for the new method compared with the standard Runge- Kutta algorithm. Animations are created to show 3-D visualizations of the torsional deformation of the cylinder.

- **Title.** *Statistical methods for rough QTL analysis* (2003)

  Students. Mark J. Giganti, University of Missouri and Nathan A. Johnson, College of William and Mary

  Mentor. Steven Garren

  Abstract. Most methods for the identification of quantitative trait loci (QTLs) employ a parametric approach which assumes the normality of the phenotype distribution. However, most applications of QTL analysis do not have normally distributed phenotypes. In this research, we compare different parametric and nonparametric approaches to both single marker and paired marker selection analysis. A simulation study is conducted to compare the prediction errors for each method.

- **Title.** *Notes on the structure of $P\Sigma_n$* (2002)

Students. Erin Corman, Keene State College and Rebecca Dolphin, Mary Washington College

Mentor. Leonard Van Wyk

Abstract. The pure symmetric automorphism group, $P\Sigma_n$, consists of those automorphisms of the free group on n generators that take each standard generator to a conjugate of itself. We give presentations for kernels of homomorphisms $P\Sigma_n \to \mathbb{Z}$ where each standard generator is sent to either 0 or 1, and provide explicit generators (as words in the standard generators) when those kernels are finitely generated. In addition, we provide recursive constructions of the defining graphs of the graph groups associated with $P\Sigma_n$.

- **Title.** *The Modified Picard-Padé Approximation Method for Singular Nonlinear Boundary Value Problems* (2002)

Students. Todd Svitzer, James Madison University and Jeff Evey, James Madison University

Mentors. Debra Warne and Paul Warne

Abstract. Many predominant numerical algorithms used to approximate solutions of nonlinear boundary-value problems (BVPs) have a Runge-Kutta foundation. A shooting algorithm using a foundation of the Picard method can potentially produce better accuracy in less time near a singularity. This also produces an effective numerical technique for BVPs, as it numerically generates and stores the coefficients of the Taylor polynomial of the solution at each step for each term of the series, using a simple progression of Cauchy products. The Taylor coefficients are then used to numerically create the coefficients of a rational polynomial Padé approximation to the solution at each step for singular BVPs. Our modified Picard-Padé algorithm allows for a smaller number of steps as the solution marches toward the singularity and provides a simple manner in which to increase (or decrease) the order of the algorithm during the computation, resulting in general in a more accurate solution nearby and at the singularity. We first develop the method theoretically and then demonstrate it for a singular test problem which is compared against a standard Runge-Kutta procedure. Singular nonlinear BVPs modeling cavitation (void formation in solids) in finite elasticity are also examined numerically.

- **Title.** *Asymptotic analysis of finite deformation in a nonlinear transversely isotropic incompressible hyperelastic half-space subjected to a tensile point load* (2002)

Student. Ethan Coon, University of Rochester

Mentors. Debra Warne and Paul Warne

Abstract. The Boussinesq problem, that is, determining the deformation in a hyperelastic half-space due to a point force normal to the boundary, is an important problem of engineering, geomechanics, and other fields to which elasticity theory is often applied. While linear solutions produce useful Green's functions, they also predict infinite displacements and other physically inconsistent results nearby and at the point of application of the load where the most critical and interesting material behavior occurs.

To illuminate the deformation due to such a load in the region of interest, asymptotic analysis of the nonlinear Boussinesq problem has been considered in the context of isotropic hyperelasticity. Studies considering transversely isotropic materials have also been broadly used in the linear theory, but have not been treated within the nonlinear framework. In this paper we extend the nonlinearly elastic isotropic analysis to transverse isotropy, producing a more general theory which also better encompasses applications involving layered media. The governing equations for nonlinearly elastic, transversely isotropic solids are derived, conservation laws of elastostatics are invoked, asymptotic forms of the deformation solutions are hypothesized, and the differential equations governing deformation near the point load are determined. The analysis also develops sequences of simple tests to determine if a transversely isotropic material can possibly sustain a finite deflection under the point load. The results are applied to a variety of transversely isotropic materials, and the effects of the anisotropy considered is demonstrated by comparison of the resulting deformation with similar asymptotic solutions in the isotropic theory.

- **Title.** *Improved estimation of location parameters for Laplace and geometric distributions under order restrictions* (2002)

  Students. Elizabeth R. Hume, Longwood University and Glen R. Leppert, James Madison University

  Mentor. Steven Garren

  Abstract. Suppose a tree order restriction is assumed on the location parameters of a Laplace distribution, such that the populations are independently sampled. We develop an isotonic regression estimator which improves on the unrestricted estimator of the smallest location parameter in terms of mean squared error. However, the isotonic regression estimator fails to stochastically dominate the unrestricted estimator, as illustrated by a counterexample. Similar results are shown for estimating the unknown parameter of a geometric distribution under order restrictions.