Titles, Abstracts, and Participants of Projects of Award 0097451

Title. *Notes on the structure of $P\Sigma_n$* (2002)
Students. Erin Corman, Keene State College
Rebecca Dolphin, Mary Washington College
Mentor. Leonard Van Wyk
Abstract. The pure symmetric automorphism group, $P\Sigma_n$, consists of those automorphisms of the free group on n generators that take each standard generator to a conjugate of itself. We give presentations for kernels of homomorphisms $P\Sigma_n \to \mathbb{Z}$ where each standard generator is sent to either 0 or 1, and provide explicit generators (as words in the standard generators) when those kernels are finitely generated. In addition, we provide recursive constructions of the defining graphs of the graph groups associated with $P\Sigma_n$.

Title. *The Modified Picard-Pade Approximation Method for Singular Nonlinear Boundary Value Problems* (2002)
Students. Todd Svitzer, James Madison University
Jeff Evey, James Madison University
Mentors. Debra Warne, Paul Warne
Abstract. Many predominant numerical algorithms used to approximate solutions of nonlinear boundary-value problems (BVPs) have a Runge-Kutta foundation. A shooting algorithm using a foundation of the Picard method can potentially produce better accuracy in less time near a singularity. This also produces an effective numerical technique for BVPs, as it numerically generates and stores the coefficients of the Taylor polynomial of the solution at each step for each term of the series, using a simple progression of Cauchy products. The Taylor coefficients are then used to numerically create the coefficients of a rational polynomial Pade approximation to the solution at each step for singular BVPs. Our modified Picard-Pade algorithm allows for a smaller number of steps as the solution marches toward the singularity and provides a simple manner in which to increase (or decrease) the order of the algorithm during the computation, resulting in general in a more accurate solution nearby and at the singularity. We first develop the method theoretically and then demonstrate it for a singular test problem which is compared against a standard Runge-Kutta procedure. Singular nonlinear BVPs modeling cavitation (void formation in solids) in finite elasticity are also examined numerically.

Title. *Asymptotic analysis of finite deformation in a nonlinear transversely isotropic incompressible hyperelastic half-space subjected to a tensile point load* (2002)

Student. Ethan Coon, University of Rochester

Mentors. Debra Warne, Paul Warne

Abstract. The Boussinesq problem, that is, determining the deformation in a hyperelastic half-space due to a point force normal to the boundary, is an important problem of engineering, geomechanics, and other fields to which elasticity theory is often applied. While linear solutions produce useful Green's functions, they also predict infinite displacements and other physically inconsistent results nearby and at the point of application of the load where the most critical and interesting material behavior occurs. To illuminate the deformation due to such a load in the region of interest, asymptotic analysis of the nonlinear Boussinesq problem has been considered in the context of isotropic hyperelasticity. Studies considering transversely isotropic materials have also been broadly used in the linear theory, but have not been treated within the nonlinear framework. In this paper we extend the nonlinearly elastic isotropic analysis to transverse isotropy, producing a more general theory which also better encompasses applications involving layered media. The governing equations for nonlinearly elastic, transversely isotropic solids are derived, conservation laws of elastostatics are invoked, asymptotic forms of the deformation solutions are hypothesized, and the differential equations governing deformation near the point load are determined. The analysis also develops sequences of simple tests to determine if a transversely isotropic material can possibly sustain a finite deflection under the point load. The results are applied to a variety of transversely isotropic materials, and the effects of the anisotropy considered is demonstrated by comparison of the resulting deformation with similar asymptotic solutions in the isotropic theory.

Title. *Improved estimation of location parameters for Laplace and geometric distributions under order restrictions* (2002)

Students. Elizabeth R. Hume, Longwood University, Farmville, Virginia
Glen R. Leppert, James Madison University

Mentor. Steven Garren

Abstract. Suppose a tree order restriction is assumed on the location parameters of a Laplace distribution, such that the populations are independently sampled. We develop an isotonic regression estimator which improves on the unrestricted estimator of the smallest location parameter in terms of mean squared error. However, the isotonic regression estimator fails to stochastically dominate the unrestricted estimator, as illustrated by a counterexample. Similar results are shown for estimating the unknown parameter of a geometric distribution under order restrictions.