Abstracts of Past JMU REU Projects

- **Title.** *Partial Metric Spaces: Representation and Classification.* (2014)

  Students. Shaun Benvie (University of Connecticut) and Owen Hill (William and Mary)

  Mentor. Elizabeth Brown

  Abstract. Partial metrics generalize the standard notion of distance, a metric, to allow for non-zero self-distances. We show how partial metrics can be constructed from standard metrics and vice versa, which gives rise to a useful representation theorem. We also analyze separation axioms for partial metric topologies, which are not in general metrizable.

- **Title.** *Approaches to Addressing Overfitting in Averaging Classifiers.* (2014)

  Students. James Munyon (Youngstown State) and Rebecca Thiem (Eastern Kentucky)

  Mentor. Lihua Chen

  Abstract. The goal of statistical classification is to learn a model with known data that will accurately predict the class of new cases. Bayesian Model Averaging is theoretically the optimal method for combining learned models, but its application in machine learning remains an open problem due to its tendency to overfit. Overfitting refers to the weight being concentrated on a single or a few models, and results from the exponential form of the weighting function. In this project, we explore a number of alternative weighting schemes in averaging decision trees that aim to address this problem. Our studies show that uniform weighting of models yields better results than Bayesian Model Averaging, but does not perform as well when compared to Random Forest and three new methods that take model performance into account.

- **Title.** *Measures of Predictor Variable Importance in Logistic Regression: A Comparative Study.* (2014)

  Students. Ranil Weerackoon (Cal State Long Beach) and Dmitry Suvorov (The College of New Jersey)

  Mentor. Dinesh Sharma

Abstract. Assessing the relative importance of predictor variables in a multiple linear regression model has often been desired by researchers when explanatory aspects of the model are sought after. A number of indices of relative importance in multiple linear regression have been proposed in literature including the squared zero-order correlation, squared standardized regression coefficient, product measure (Pratts index), the squared standardized regression coefficient of regressed on the least squares orthogonal of j weight measure. In this study we extend these indices to the case of logistic regression model, when the dependent variable takes a binary form. A Monte Carlo Study is conducted to compare the performance of these indices numerically. Recommendations are made as to which of the relative importance indices are effective for ranking the predictors.

- **Title.** *Fluid Dynamics Around Swimming Worms: From Theory to Experiment.* (2014)

  Students. Jonathan Gutierrez (St. Mary's (TX)) and Megan Sorenson (Concordia University)

  Mentor. Eva Strawbridge

  Abstract. C. elegans have been extensively researched regarding locomotion. However, most mathematical studies have focused on body dynamics rather than the fluid. As the nematodes undulate in a sinusoidal fashion, they cause fluid movement that has been studied experimentally but not modeled computationally on this scale. Utilizing the Navier-Stokes equation, regularized stokeslets, and the method of images, we computed the dynamics of the surrounding fluid. Our results strikingly matched experimental outcomes in various ways, including the distance particles travelled in one period of undulation, as well as qualitatively and quantitatively matching velocity fields. We then implemented this method using video data of C. elegans swimming and successfully reproduced the fluid dynamics. This is a novel application of the method of regularized stokeslets that combines theory and experiment. We expect this approach to provide insight in generating hypotheses and informing experimental design.

- **Title.** *The Power Series Method and Applications.* (2013)

  Students. Stephanie Brown (SUNY Buffalo) and Brandon Alexander (University of Maryland Baltimore County)

Mentors. Steve Lucas and Jim Sochacki

Abstract. We will discuss the method of solving ODEs known as the Power series method. This method allows us to solve systems of equations to arbitrary accuracy. Our focus has been applying this method to a system of equations representing an autocatalytic reaction as well as to various stochastic differential equations.

- **Title.** *Using PSM to Study Delay Differential Equations with Chaos.* (2013)

  Students. Luke Edwards (Pennsylvania State University) and Ben Weidenaar (Providence College)

  Mentors. Steve Lucas and Jim Sochacki

  Abstract. We use PSM (for the first time as far as we know) to solve delay differential equations (DDEs). We show how PSM has to be modified to handle delays and demonstrate the efficacy and robustness of the algorithm on some examples. In particular, we apply the PSM to the pursuit problem with and without delay and compare with other methods. We show that PSM preserves the properties of the pursuit problem more accurately. We demonstrate chaos for the pursuit problem through the time delay.

- **Title.** *One-Singular Knots I.* (2013)

  Students. Sarah Nicholson (University of Dallas) and Katie Tucker (Willamette University)

  Mentors. Laura Taalman and Leonard Van Wyk

  Abstract. We describe the three methods we used to create knots with a lone singularity, methods we used to distinguish these one-singular knots, and surprising difficulties encountered along the way.

- **Title.** *One-Singular Knots II.* (2013)

  Students. David Brown (Grinnell College) Ryan Stees (JMU)

  Mentors. Laura Taalman and Leonard Van Wyk

  Abstract. We explore one of the difficulties alluded to in the previous talk: the dependence of the one-singular set on the projection of a classical knot, focusing specifically on minimal projections and on torus knots.

- **Title.** *A New Proof of the Lecture Hall Theorem.* (2012)

Students. Meredith Harris (Clemson University) and Alex Komarinski (Arizona State University)

Mentors. Brant Jones and Edwin O'Shea

Abstract. Euler proved a bijection between integer partitions with distinct parts and integer partitions with odd parts. The lecture hall partitions can be thought of as a way to arrange the levels of seats in a lecture hall so every student may see the speaker. Bousquet-Melou and Eriksson provided a proof that there is a bijection between these lecture hall partitions and integer partitions of "small" odd parts, that generalizes Euler's celebrated result. In this presentation we provide a new Coxeter-free bijection using abacus diagrams and bounded partitions.

- **Title.** *New Results on the Coxeter Group Structure of Lecture Hall Partitions.* (2012)

  Students. Laura Bradford (Bard College) and Carly Matson (University of Virginia)

  Mentors. Brant Jones and Edwin O'Shea

  Abstract. Bousquet-Melou and Eriksson interpreted lecture hall partitions in terms of the affine Coxeter group of type C. We extend this interpretation to affine type B. We also use this framework to study a related structure called the d-sequence that encodes the successive differences between parts of the lecture hall partitions.

- **Title.** *Recent Progress on Rota's Basis Conjecture.* (2012)

  Students. Stephanie Bittner (Virginia Wesleyan College) and Michael Cheung (Elizabethtown College)

  Mentors. Joshua Ducey and Minah Oh

  Abstract. Suppose you are given n bases of an n-dimensional vector space. Additionally suppose that each basis is assigned a particular color: say the first basis is red, the second blue, etc. Then Rota's Basis Conjecture asserts that one can always repartition the multiset union of these bases into n "rainbow bases"–that is, each new basis will contain exactly one vector of each color.

  This innocent-looking conjecture has been open for over twenty years. In this talk we explain some of the conjecture's history, as well as

recent progress made. We also discuss a computational proof for this conjecture that generalizes to the theory of matroids.

- **Title.** *Invariants of an Incidence Matrix Related to Rota's Basis Conjecture.* (2012)

  Students. Xuyi Guo (Stanford University) and Adam Zweber (Carleton College)

  Mentors. Joshua Ducey and Minah Oh

  Abstract. Suppose you are given n disjoint sets each of size n. Form a matrix with rows and columns indexed by the collection of transversals of these n sets. Put a 1 in the (i, j)-entry if transversal i is disjoint from transversal j, otherwise put a 0.

  Aside from being an interesting combinatorial object in its own right, this zero-one matrix is very closely related to the n-dimensional case of Rota's Basis Conjecture. In this talk we discuss several numerical invariants of the matrix, in particular its eigenvalues and Smith normal form.

- **Title.** *Periodicity and asymptotics of Tchoukaillon sequences.* (2011)

  Students. David Creech (Central Michigan University) and Jeff Anway (Longwood University)

  Mentors. Laura Taalman and Anthony Tongen

  Abstract. Tchoukaillon is a single-player sowing game in which there is a unique board for each natural number, indexed by board size. We will examine the global periodicity properties of Tchoukaillon and the long-term asymptotic behavior that describes the surprisingly complicated relationship between board size and board length.

- **Title.** *Algebraic Tchoukaillon representations.* (2011)

  Students. Benjamin Warren (Ramapo College of New Jersey) and Fanya Wyrick-Flax (Bard College)

  Mentors. Laura Taalman and Anthony Tongen

  Abstract. We present three algebraic representations of the sowing game Tchoukaillon and construct maps translating between each representation. These maps yield information about the relationships between the pit size and move sequences and in addition enable us to construct a binary addition operation on Tchoukaillon boards.

- **Title.** *Permutations avoidance and the Catalan triangle.* (2011)

  Students. Wesley K. Hough (Hanover College) and Jacob W. Ziefle (The College of New Jersey)

  Mentors. Rebecca Field and Brant Jones

  Abstract. Some of the first questions regarding permutation pattern avoidance arose from sorting methods in computer science. In one-line notation, a permutation of n letters avoids a pattern of k letters if every subsequence in the permutation has at least two elements out of order relative to the pattern. It has been previously shown that the size of the set of permutations of n letters that avoid a 3-letter pattern is equal to the n-th Catalan number. In this paper, we refine this result for all patterns of size 3 by intuitively partitioning the set of permutations of n letters that avoid a given pattern and relating these partitions to some well-known refinements of the Catalan numbers.

- **Title.** *Homology of pattern avoidance.* (2011)

  Students. Derek DeSantis (California State University Channel Islands) and Rebecca Meissen (Worcester Polytechnic Institute)

  Mentors. Rebecca Field and Brant Jones

  Abstract. In this presentation, we explore the relationship between topology and partially ordered sets. We consider the symmetric group Sn under the Bruhat partial order. Bjrner and Wachs showed that any interval in Sn is homotopic to a single sphere. We examine the topology of Sn([p]), the symmetric group on n letters which avoids a specic permutation pattern p. Babson and Hersh introduced a method for analyzing the homology of a poset via Discrete Morse Theory. We use this to study Sn([321]).

- **Title.** *Applications of and Alternatives to Algorithm X for the Exact Cover Problem.* (2010)

  Students. Eddie Tu (Randolf-Macon College) and Bjorn Wastvedt (St. Olaf College)

  Mentor. Steven Lucas

  Abstract. The Exact Cover problem is NP-Complete and is generally solved by Donald Knuth's Algorithm X, an efficient tree search. Aiming to

make use of this algorithm's efficiency, we propose and investigate methods to convert several other NP-Complete problems to the Exact Cover problem. In this context, we discuss the Graph Coloring problem, the Hamiltonian Cycle problem, and the 0-1 Programming problem. The last case suggests a new alternative to Algorithm X for solving Exact Cover problems: We show how simple linear algebra can often reduce the size of an Exact Cover problem significantly. We use Sudoku, a popular Exact Cover problem, as a test case and motivation throughout alongside more general examples.

- **Title.** *Using Packets to Create Solving Symmetries in Sudoku.* (2010)

  Students. Malcom Rupert (Western Washington University) and Harrison Chapman (Bowdoin College)

  Mentor. Elizabeth Arnold

  Abstract. Recently there has been a lot of interest in the mathematics of the popular game Sudoku. In a typical Sudoku puzzle, a number of initial clues are given, and the solver uses strategies to fill in the remaining clues to complete the board. A well-known open problem is, How many initial clues are necessary for the puzzle to have a unique completion? In this talk, we shift the focus of study from clues to what we call packets. A packet gives information about what clues can NOT be in a cell. Introducing packets gives rise to some interesting questions about Sudoku and its 4 ? 4 counterpart, Shidoku. One such question is what is the minimum number of packets needed to describe a puzzle with a unique completion? This question parallels the minimum clue question. Packets are also intimately related to the Boolean system of polynomial equations used to describe the constraints of a Sudoku puzzle. They can be used to more efficiently calculate a Grobner basis of the ideal generated by this system of equations. Packets are also inherently related to human methods for solving Sudoku puzzles. To emulate human solving strategies we introduce the idea of solving symmetries functions which manipulate a puzzle while maintaining the same solutions. We show that these solving symmetries form a group which acts on the set of Sudoku puzzles.

- **Title.** *Estimating Variance-Mean Mixtures of Normals.* (2010)

  Students. Sam Helmich (Winona State University), Kevin Stoll (Baldwin-Wallace College), Holly Gardner (Loyola University), Caitlin Steiner

(College of William & Mary)

Mentor. Hasan Hamdan and Ling Xu

Abstract. In this presentation, we will introduce a new method, NVM_UNMIX for estimating the density function of normal variance-mean mixtures. This new method is a manipulation of the previous developed normal scale mixtures program UNMIX (Hamdan et al., 2005). NVM_UNMIX is designed to model normal variance-mean mixtures by minimizing the weighted square distance between an empirical density and the theoretical mixture, taking into account any factors that effect the variability of the estimates. This modeling technique is then evaluated using several simulated examples and is compared to the Bayesian approach in a couple of real life situations.

- **Title.** *What Moves You: Using Legs for Vehicular Transportation.* (2009)

  Students. Jonathan Graf (Towson University) and Olga Stulov (SUNY New Paltz)

  Mentor. Jim Sochacki

  Abstract. Most vehicles are transported by the rotation of wheels. The Department of Mathematics and Statistics and Department of Engineering are interested in developing vehicles that will be driven by the motion of legs rather than wheels. In this talk we discuss the motion of five different legs: first, we derive the equations of motion for each leg; second, we calculate the equations for velocity, acceleration, energy and power; third, we optimize the motion by minimizing energies and forces. In order to obtain these results, we developed a differential equation, solved it using the Parker-Sochacki Method and reached the optimal solution using Maples minimization package.

- **Title.** *Diversions: Sudoku, Shidoku, and ... Grobner Bases? An Algebraic & computer Systems Approach to Counting Boards* (2009)

  Students. Katharina Carella (Ithaca College) and Matt Menickelly (Miami University)

  Mentor. Elizabeth Arnold

  Abstract. We investigate various counting proofs for Shidoku boards and related variants, such as the number of possible solution boards from incomplete puzzles. We also look into the algebraic group derived

from symmetries of Shidoku boards. We use these group isomorphisms to classify all possible numbers of solutions from incomplete puzzles. We use Grobner Basis representations of Shidoku and Sudoku to obtain these results. We provide a complete classification of all the possible number of solutions that can result from incomplete Shidoku puzzles.

- **Title.** *Improved One-Sample Confidence Interval and Multiple Comparisons of Binomial Proportions* (2009)

  Students. Kristin Haldeman (Cal State Long Beach) and Christopher Tait (Hampden-Sydney College)

  Mentor. Kane Nashimoto

  Abstract. We study confidence intervals and tests of hypotheses involving binomial proportions. In the first part of the study, we examine one-sample confidence intervals. The commonly used Wald interval suffers severe undercoverage. The Score interval performs more favorably. The Agresti and Coull interval (1998), which uses the "add 2 method," shows overcoverage. We propose a modification of Agresti and Coull that has more uniform coverage probabilities over a wide range of true proportions. In the second part of the study, we consider comparisons of k independent proportions. In this context, Agresti et al. (2008) show that pairwise comparisons using the Studentized range distribution work better than the comparisons using the Wald or Score methods with Bonferroni adjustments. We propose a new 2-stage method of multiple comparisons (global test followed by pairwise comparisons). Simulation results show that our method is more powerful than most of the existing methods and that it keeps the familywise error rate near the nominal level.

- **Title.** *Distance Functions and Attribute Weighting in a k-Nearest Neighbors Classifier with an Ecological Application* (2009)

  Students. Alylssa Frazee (St. Olaf College) and Matthew Hathcock (Winona State University)

  Mentor. Samantha Prins

  Abstract. To assess environmental health of a stream, field, or other ecological "object," characteristics of that object should be compared to

a set of reference objects known to be healthy. Using streams as "objects," we propose a k-nearest neighbors algorithm (Bates Prins and Smith, 2006) to find the appropriate set of reference streams to use as a comparison set for any given test stream. Previously, investigations of the k-nearest neighbors algorithm have utilized a variety of distance functions, the best of which has been the Interpolated Value Difference Metric (IVDM), proposed by Wilson and Martinez (1997). We propose two alternatives to the IVDM: Wilson and Martinez's Windowed Value Difference Metric (WVDM) and the Density-Based Value Difference Metric (DBVDM), developed by Wojna (2005). We extend the WVDM and DBVDM to handle continuous response variables and compare these distance measures to the IVDM within the ecological k-nearest neighbors context. Additionally, we compared two existing attribute weighting schemes (Wojna 2005) when applied to the IVDM, WVDM, and DBVDM, and we propose a new attribute weighting method for use with these distance functions as well. In assessing environmental impairment, the WVDM and DBVDM were slight improvements over the IVDM. Attribute weighting also increased the effectiveness of the k-nearest neighbors algorithm in this ecological setting.

- **Title.** *Higher Order Tensor Operations and Their Applications* (2008)

  Students. Scott Ladenheim (Syracuse University) and Emily Miller (The College of New Jersey)

  Mentor. Carla Martin

  Abstract. A tensor, which can be considered as n-dimensional arrays of data can be manipulated in a similar manner to matrices, which are considered to be 2-dimensional data arrays. We define the analogous operations of addition, multiplication, invertibility and transposition for tensors in such a way that the set of $n \times n \times n$ invertible tensors form a group under addition and multiplication. From these properties, we develop the definition of the Singular Value Decomposition (SVD) for a tensor. We utilize the tensor SVD in two applications: The first uses the tensor SVD to obtain a low rank approximation of a video file. This results in a file of reduced size, but without a loss in the original video's quality. The second application is a handwritten digit recognition algorithm. When given a sample digit, the algorithm

recognizes and returns the correct value of the input digit.

- **Title.** *Numerical Evidence on the Uniform Distribution of Power Residues for Elliptic Curves* (2008)

  Students. Jeffrey Hatley (The College of New Jersey) and Amanda Hittson (Bryn Mawr College)

  Mentor. Jason Martin

  Abstract. Elliptic curves, which lie in the intersection of number theory, algebra, and geometry, are the subject of much research due to several amazing properties they possess. This talk develops the basic theory of elliptic curves and presents an interesting theorem relating a certain family of elliptic curves and the distribution of prime numbers. New data, which supports the veracity of an analagous conjecture and which was collected during the summer REU program, is also presented.

- **Title.** *Exploring Male Dimorphism in the Dung Beetle using a Discrete-time Stochastic Population Model* (2008)

  Students. Phillip Andreae (Emory University), Adam Falk (Grand Valley State University), Theresa Klinkhammer (Saint Mary's College) and Sarah Mecholsky (Agnes Scott College)

  Mentors. Anthony Tongen and Brian Walton

  Abstract. Certain dung beetle species that belong to the genus Onthophagus display an interesting behavioral and physical male dimorphism, with populations split between horned and hornless males with implications for reproductive behavior. We have developed several variations of a discrete time stochastic population model to study this male dimorphism and determine if there exists an optimal strategy for dividing a population between horned and hornless males. By comparing two subpopulations, each of which is characterized by a strategy, we can determine the probability that an individual in the next generation would belong to a particular subpopulation and inherit the corresponding particular strategy. We will present our results that show that we can find an optimal strategy using adaptive dynamics for varying encounter rates, sperm potency, and body size.

- **Title.** *Bayesian Multiple Comparisons for Treatments with a Control* (2007)

Students. Christopher R. Dienes (Montana Tech University) and Dustin B. Hevener (JMU)

Mentor. Kane Nashimoto

Abstract. Comparison of k treatment means with a control mean is considered. In this setting, the pioneering work by Dunnett is well known. We propose both two-sided and one-sided multiple-comparison procedures using a Bayesian hierarchical model. We parameterize each treatment mean as the control mean plus the effect due to the treatment. The effect parameter has a mixture prior with a point mass at zero and a continuous density. For each pairwise comparison, inference about mean inequality is based on the posterior probability of no effect. Though the proposed method is more conservative than the frequentist counterpart, it flexibly accommodates heterogeneity of variances and unequal sample sizes. Further, the model can be utilized in many areas of applied research, where treatments vs. control is of primary interest.

- **Title.** *An Evaluation of Hidden Markov Models for the Analysis of Binding Events in Proteins* (2007)

  Students. Rachel Louise Bayless (Wheaton College) and Alexandra Noelani Menton (University of Maryland, Baltimore County)

  Mentor. D. Brian Walton

  Abstract. Researchers have developed various experimental methods to better understand ligand-receptor binding. We provide a means of joining a statistical model with an experiment to assess how well a model, given parameter values, describes the experimental observations. Our models estimate on and off rates for a dumbbell assay of actomyosin binding and construct the sequence of binding events between actin and myosin that best matches the observations. We expand upon the analysis of the variance hidden Markov method, first implemented by Smith et al., 2001, which considers the variance of observations over windows of data rather than the original displacements. We then introduce a more detailed analysis that operates directly on the displacement record. This model utilizes all possible given information and thus yields a more detailed description of binding events. Finally, we compare the performance of these models using simulated data.

- **Title.** *Alexander Polynomials of Central Braid Knots* (2007)

  Students. Nathan Brothers (UNC Asheville) and Sean Evans (Millersville University)

  Mentors. Laura Taalman and Leonard Van Wyk

  Abstract. Central braid knots are a new class of periodic knots. The talk will start with a brief introduction to knot theory and then discuss properties of central braid knots, including their crossing matrices, Alexander matrices, and Alexander polynomials.

- **Title.** *The Genus and m-alternating Excess of Central Braid Knots* (2007)

  Students. Debra Witczak (Benedictine University) and Carolyn Yarnall (University of San Diego)

  Mentors. Laura Taalman and Leonard Van Wyk

  Abstract. We define a new class of braid knots called central braid knots. Using the degree of the Alexander polynomial, we provide a formula for the genus and a bound on the crossing number for any central braid knot. The crossing number bound leads to a bound on the m-alternating excess of central braid knots.

- **Title.** *Analysis of a Lotka-Volterra Competition Model with a Non-Linear Relationship Between Spaces* (2005)

  Students. Austin Taylor (University of Alabama) and Amy Vess (JMU)

  Mentor. James Liu

  Abstract. We consider a modified Lotka-Volterra competition model, which incorporates a non-linear relationship representing the interaction between species. The qualitative properties of this system are studied and compared to the qualitative properties of the classic Lotka-Volterra competition equations. We obtain results suggesting that these qualitative properties are similar with slight differences allowing the modified model to be a better representation of some biological situations.

- **Title.** *The Isoperimetric Numbers of Block Design Graphs* (2005)

  Students. Christopher Miller (Fairfield University) and Amber Russell (Mississippi State University)

  Mentor. Jason Rosenhouse

Abstract. The isoperimetric number of a graph is a measure of the graph's connectedness. It finds numerous applications in mathematics and computer science. In this talk, we will obtain new upper and lower bounds on the isoperimetric numbers for graphs associated to block designs. We will focus particularly on the special cases of finite projective and affine planes.

- **Title.** *Comparing Ratio Estimators Based on Systematic Samples* (2005)

  Students. Amy Jacks (University of Missouri - Rolla) and John Szarka (JMU)

  Mentor. Hasan Hamdan

  Abstract. The purpose of this study is to evaluate two competitive ratio estimators, often called the mean of ratios and ratio of means, when a systematic sample of size n with a random start is used. For example, in stereology this might be the ratio by volume of mitochondria in a liver cell or the proportion of a mineral in a sample of a rock. Since there is no explicit formula for the variance of a ratio estimator in such cases, several current variance estimators are presented and compared using simulated objects. Two new approaches are also suggested. The first new method is basically a bootstrap estimate using a non-linear additive regression technique. A Monte-Carlo simulation is done using the predicted values from the fitted model to find estimates for the variances. The second method is based on finding the best linear unbiased estimator of the slope assuming a non- constant variance. These new approaches seem to have high accuracy and to behave very well in terms of variance and bias or mean square error.

- **Title.** *Size and Difficulty of Mass Problems* (2005)

  Students. Justin Palumbo (Rutgers University) and Andre Kornell (Princeton University)

  Mentor. Elizabeth Brown

  Abstract. Muchnik and Medvedev reducibility are equivalence relations between mass problems, or sets of functions, of similar computational difficulty. In this talk we will discuss notions of the size of a mass problem, namely the properties of smallness, cardinality, and a new property, d-smallness. We will present some results on the relation-

ship between the difficulty of a mass problem and its size under these notions.

- **Title.** *p-Coloring Classes of Torus Knots* (2004)

  Students. Anna-Lisa Breiland, Willamette College and Layla Oesper, Pomona College

  Mentor. Laura Taalman

  Abstract. We develop a theorem for determining the p-colorability of any (m,n) torus knot. We also prove that any p-colorable (m,n) torus knot has exactly one p-coloring class. Finally, we show that every p-coloring of the braid projection of an (m,n) torus knot must use all of the p colors.

- **Title.** *Analysis for Intracranial Saccular Aneurysms: I. Nonlinear anisotropic biomechanical model, II. Fast, adaptive, and accurate computational algorithm, and III. scientific visualization.* (2004)

  Students. James Collins, James Madison University and Matthew Watts, James Madison University

  Mentor. Paul Warne

  Abstract. Biomechanical modeling has great potential to contribute to and improve the requisite diagnostic and therapeutic capabilities of health care. Arterial wall mechanics is particularly crucial not only for the understanding of blood circulatory physiology but also for analyzing the mechanisms of vascular disorders. Nonlinear continuum mechanics is an excellent framework for formulating mathematical models that can accurately capture the complex behavior of biological soft tissues. Among other vascular disorders, intracranial saccular aneurysms have received attention as an open problem that merits increased research. A simple dynamic deformation studied by the continuum biomechanics community is one of uniaxial stretch in the radial direction. Using the approximations of membrane theory for example, Humphrey et al. study this problem assuming the wall of the sphere to be infinitesimally small. Instead, we study this problem for a finite, thin-walled sphere using the fully three-dimensional exact governing equations of nonlinear elasticity. The dynamics of our intracranial saccular aneurysm model are compared for three different material models: a classic neo- Hookean elastic strain energy, a simple Fung strain

energy often used in modeling biological tissues, and a new strain energy capturing anisotropy of a radially fiber-reinforced material that suggests ways to control the growth of the aneurysm. Such modeling of physical and biological problems generally leads to complicated ODE-BVPs, unable to be solved analytically. Thus, accurate, efficient, and adaptable numerical methods are critical, especially since singularities and bifurcation phenomena are common. As accuracy can be significantly compromised near a singularity for many predominant nonlinear ODE numerical solution algorithms (e.g., Runge-Kutta based methods), here we also incorporate a tractable new algorithm (Algebraic-Maclaurin-Padé (AMP)) that approximates the solution to an ODE system by the Padé approximant at each step, and can produce much better accuracy in far less time near a singularity. We also develop adaptive time-stepping for the AMP method, using the error calculated from the Padé approximant, allowing for shorter calculation time while maintaining the same accuracy as with fixed time-stepping. Finally, we develop animations of the aneurysm dynamics from our computed numerical data.

- **Title.** *One Regularity of Cayley graphs* (2004)

  Students. Mary Balmes, St. Joseph's College and Jaclyn Kaminski, Xavier University

  Mentor. Jason Rosenhouse

  Abstract. The problem of finding highly symmetric graphs is almost as old as graph theory itself. We considered the problem of finding examples of one-regular Cayley graphs, meaning they are arc-transitive and each arc has a trivial arc-stabilizer. The first known cubic one-regular graph was found in 1952 by Frucht, and we examined his construction in detail. More recently Kwak and Oh found a method for constructing one- regular Cayley graphs of any even valency, and we considered their method as well. Examples of one-regular Cayley graphs of odd valency are harder to come by. We discovered several properties any such graph must satisfy. We also considered direct and semi-direct products of groups, and discovered certain properties such groups must have if they are to give rise to one-regular Cayley graphs.

- **Title.** *Using Scale Mixtures of Normals to Model Continuously Compounded Return* (2004)

Students. Kristen Dardia, James Madison University and Melanie Wilson, Allegheny College

Mentor. Hasan Hamdan

Abstract. In this paper, a new method for es timating the parameters of scale mixtures of normals is presented. The new method is called UNMIX and is based on minimizing the weighted square distance between exact values of the density of the scale mixture and estimated values using kernel smoothing techniques over a prespecified grid of x-values and a grid of potential sigma values. Applications of the method are made in modeling the continuously compounded return of stock prices. Modeling this ratio with UNMIX proves promising in comparison with other existing techniques that use only one normal component, or those that use more than one component based on the EM algorithm as the method of estimation.

- **Title.** *Pretzel knots and colorability* (2003)

  Students. Kathryn Brownell, Lenoir-Rhyne College and Kaitlyn O'Neil, Merrimack College

  Mentor. Laura Taalman

  Abstract. We develop a formula for determining the number of fundamentally different ways that an m-colorable knot can be m-colored, based on the m-nullity of the knot. We then determine the m-nullity of any (p,q,r) pretzel knot, and thus a way to determine the m-colorability and number of fundamentally different m-colorings of any pretzel knot.

- **Title.** *k-alternating knots* (2003)

  Students. Philip Hackney, Central Michigan University and Nathan Walters, Drake University

  Mentor. Leonard Van Wyk

  Abstract. A projection of a knot is *k-alternating* if its overcrossings and undercrossings alternate in groups of k as one reads around the projection (an obvious generalization of the notion of an alternating projection). More generally, a projection is *w-repeating* if, rather than alternating in groups of k, the crossings follow any pattern whatsoever. We show every knot that admits a k-alternating projection also admits

a $(k+1)$-alternating projection, and that every knot that admits a w-repeating projection admits a k-alternating projection for $k \leq \frac{|w|}{2}$. We also prove the surprising result that every knot admits a 2-alternating projection, which partitions nontrivial knots into two classes: alternating and 2-alternating. Finally, we explore the notion of the *k-alternating excess* of a knot, the difference between the smallest number of crossings in a k-alternating projection of the knot and its crossing number.

- **Title.** *Padé approximates for torsion of a compressible nonlinearly elastic cylinder: modeling, computation, and visualization* (2003)

  Students. Danielle Miller, James Madison University and Jennifer Salyer, East Tennessee State University

  Mentors. Debra Warne and Paul Warne

  Abstract. The boundary-value problem (BVP) resulting from the equations of nonlinear elastostatics for torsion of a circular cylinder for a class of general Blatz-Ko materials involves a nonlinear, singular, 2^{nd} order ordinary differential equation for the radial deformation with nonstandard boundary conditions. Using a new computational approach to project the BVP to a polynomial system, we are able to compute the MacLaurin coefficients of the solution to any degree and then accurately solve for the radial deformation by numerically creating its Padé approximation. Significant improvements in both accuracy and efficiency are shown for the new method compared with the standard Runge- Kutta algorithm. Animations are created to show 3-D visualizations of the torsional deformation of the cylinder.

- **Title.** *Statistical methods for rough QTL analysis* (2003)

  Students. Mark J. Giganti, University of Missouri and Nathan A. Johnson, College of William and Mary

  Mentor. Steven Garren

  Abstract. Most methods for the identification of quantitative trait loci (QTLs) employ a parametric approach which assumes the normality of the phenotype distribution. However, most applications of QTL analysis do not have normally distributed phenotypes. In this research, we compare different parametric and nonparametric approaches to both single

marker and paired marker selection analysis. A simulation study is conducted to compare the prediction errors for each method.

- **Title.** *Notes on the structure of PΣ_n* (2002)

  Students. Erin Corman, Keene State College and Rebecca Dolphin, Mary Washington College

  Mentor. Leonard Van Wyk

  Abstract. The pure symmetric automorphism group, PΣ_n, consists of those automorphisms of the free group on n generators that take each standard generator to a conjugate of itself. We give presentations for kernels of homomorphisms P$\Sigma_n \to \mathbb{Z}$ where each standard generator is sent to either 0 or 1, and provide explicit generators (as words in the standard generators) when those kernels are finitely generated. In addition, we provide recursive constructions of the defining graphs of the graph groups associated with PΣ_n.

- **Title.** *The Modified Picard-Padé Approximation Method for Singular Nonlinear Boundary Value Problems* (2002)

  Students. Todd Svitzer, James Madison University and Jeff Evey, James Madison University

  Mentors. Debra Warne and Paul Warne

  Abstract. Many predominant numerical algorithms used to approximate solutions of nonlinear boundary-value problems (BVPs) have a Runge-Kutta foundation. A shooting algorithm using a foundation of the Picard method can potentially produce better accuracy in less time near a singularity. This also produces an effective numerical technique for BVPs, as it numerically generates and stores the coefficients of the Taylor polynomial of the solution at each step for each term of the series, using a simple progression of Cauchy products. The Taylor coefficients are then used to numerically create the coefficients of a rational polynomial Padé approximation to the solution at each step for singular BVPs. Our modified Picard-Padé algorithm allows for a smaller number of steps as the solution marches toward the singularity and provides a simple manner in which to increase (or decrease) the order of the algorithm during the computation, resulting in general in a more accurate solution nearby and at the singularity. We first develop the method theoretically and then demonstrate it for a singular

test problem which is compared against a standard Runge-Kutta procedure. Singular nonlinear BVPs modeling cavitation (void formation in solids) in finite elasticity are also examined numerically.

- **Title.** *Asymptotic analysis of finite deformation in a nonlinear transversely isotropic incompressible hyperelastic half-space subjected to a tensile point load* (2002)

  Student. Ethan Coon, University of Rochester

  Mentors. Debra Warne and Paul Warne

  Abstract. The Boussinesq problem, that is, determining the deformation in a hyperelastic half-space due to a point force normal to the boundary, is an important problem of engineering, geomechanics, and other fields to which elasticity theory is often applied. While linear solutions produce useful Green's functions, they also predict infinite displacements and other physically inconsistent results nearby and at the point of application of the load where the most critical and interesting material behavior occurs. To illuminate the deformation due to such a load in the region of interest, asymptotic analysis of the nonlinear Boussinesq problem has been considered in the context of isotropic hyperelasticity. Studies considering transversely isotropic materials have also been broadly used in the linear theory, but have not been treated within the nonlinear framework. In this paper we extend the nonlinearly elastic isotropic analysis to transverse isotropy, producing a more general theory which also better encompasses applications involving layered media. The governing equations for nonlinearly elastic, transversely isotropic solids are derived, conservation laws of elastostatics are invoked, asymptotic forms of the deformation solutions are hypothesized, and the differential equations governing deformation near the point load are determined. The analysis also develops sequences of simple tests to determine if a transversely isotropic material can possibly sustain a finite deflection under the point load. The results are applied to a variety of transversely isotropic materials, and the effects of the anisotropy considered is demonstrated by comparison of the resulting deformation with similar asymptotic solutions in the isotropic theory.

- **Title.** *Improved estimation of location parameters for Laplace and geometric distributions under order restrictions* (2002)

Students. Elizabeth R. Hume, Longwood University and Glen R. Leppert, James Madison University

Mentor. Steven Garren

Abstract. Suppose a tree order restriction is assumed on the location parameters of a Laplace distribution, such that the populations are independently sampled. We develop an isotonic regression estimator which improves on the unrestricted estimator of the smallest location parameter in terms of mean squared error. However, the isotonic regression estimator fails to stochastically dominate the unrestricted estimator, as illustrated by a counterexample. Similar results are shown for estimating the unknown parameter of a geometric distribution under order restrictions.