Department of Mathematics and Statistics Colloquium

The Cognitive Gap in Mathematical Induction

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Abstract: Proof by mathematical induction poses a persistent challenge for students enrolled in proofs-based mathematics courses. Prior research indicates a number of related factors that contribute to the challenge, and suggests fruitful instructional approaches to support students in meeting that challenge. In particular, researchers have suggested quasi-induction as an intuitive approach to understanding the role of the inductive implication. However, a cognitive gap remains in transitioning to formal proof by induction. The gap includes the cognitive demand of quantification and the danger of inadvertently assuming what one is trying to prove. Informed by prior research, we designed instruction that builds from students' conceptualizations of logical implication, utilizes quasi-induction, and introduces a novel set of tasks designed to bridge the gap that remains. We studied this research-based instructional design within two sections of an Introduction to Proofs course at a large public four-year university in the southeastern United States. Our findings bring together extant literature and highlight nuances not previously reported, particularly regarding reasons students appear to conflate the assumption of the statement to be proved with the assumption of the inductive hypothesis. For instance, quantification of the inductive implication and the inductive assumption require careful use of language, using terms that may seem ambiguous to students, outside of mathematical convention.

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