Department of Mathematics and Statistics Colloquium

Thicket Density

Siddharth Bhaskar – Computer Science – James Madison University

Abstract: A *set system* is a domain X along with an arbitrary family of subsets of X---we do not impose any particular structure on this family. Set systems are of interest in various disciplines such as discrete and computational geometry, machine learning theory, and more. Various combinatorial invariants can be associated with any set system; two examples are *VC dimension* and *Littlestone dimension*. There is a precise sense in which the relationship between VC and Littlestone dimension corresponds to the relationship between learning via *non-adaptive* and *adaptive* queries.

One of the fundamental results in the combinatorics of set systems is the *SauerShelah lemma*, which says that a certain integer-valued function, the *shatter function*, grows either at most polynomially or at least exponentially depending on whether the VC dimension of the set system is finite or infinite. This has been called an "eigentheorem" for its many applications (to statistics, discrete geometry, graph theory, model theory) as well as many proofs (algebraic, combinatorial, etc.).

In this talk we shall introduce a new integer-valued function that we call the *thicket shatter function*, and show that it has exactly the same relationship to Littlestone dimension that the shatter function has to VC dimension. However, whereas the growth rate of the shatter function can assume any real value greater than 1, the growth rate of the thicket shatter function must be integervalued! This last fact does have a purely combinatorial proof (which I do not understand), but time permitting, I shall sketch a short proof that uses the notion of compactness from model theory.

Monday, April 4th at 3:10 pm via Zoom