Nonlinear Dynamical System in Humanoid Robots

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Background

- Paper was written by Jan Ijspeert, Jun Nakanishi and Stefan Schaal
- Created a Dynamical System for autonomous robotic movements
  - Models arm movements
  - Dynamical System works as a Control Policy which sends messages to the controller of the humanoid robot
  - Movement is represented in Kinematic movements
Dynamical System

\[
\begin{align*}
\dot{z} &= \alpha_z (\beta_z(g - y) - z) \\
\dot{y} &= z + \frac{\sum_{i=1}^{N} \psi_i \omega_i}{\sum_{i=1}^{N} \psi_i} \nu
\end{align*}
\]

- This system is our Control Policy
- \(g\) is our goal state
- \(Y\) is our desired state
- Constants \(\alpha_z\) and \(\beta_z > 0\)
- Describes a trajectory \(y\) towards a goal \(g\)
Dynamical System Cont.

\[
\dot{v} = \alpha_v (\beta_v (g - x) - v) \\
\dot{x} = v
\]

- This system describes the internal state of the robot
- Essential to determining the velocity
- Constants \( \alpha_z \) and \( \beta_z > 0 \)
- \( X \) is used to localize the Gaussian Kernel in the first system
- \( V \) is a scaling term used to ensure the system will have a unique attractor
Gaussian Kernel

\[
\frac{\sum_{i=1}^{N} \psi_i \omega_i}{\sum_{i=1}^{N} \psi_i} \text{ where } \psi_i = e^{-\frac{1}{2\sigma_i^2}(\bar{x} - c_i)^2}
\]

- Used for machine learning and prediction
- \(\omega_i\) found using locally weighted regression
Stability

- We have one fixed point for \((z,y)\) at \((0,g)\)

- Next we find the Jacobian Matrix:

\[
\begin{bmatrix}
-\alpha_z & -\alpha_z \beta_z \\
1 & 0
\end{bmatrix}
\]

- Then we evaluate the Jacobian Matrix at \((0,g)\) and find the eigenvalues

- Our eigenvalues are:

\[
\lambda_1 = \frac{1}{2} \left( -\sqrt{\alpha_z} \cdot \sqrt{\alpha_z - 4\beta_z} - \alpha_z \right)
\]

\[
\lambda_2 = \frac{1}{2} \left( \sqrt{\alpha_z} \cdot \sqrt{\alpha_z - 4\beta_z} - \alpha_z \right)
\]
Stability – Imaginary Eigenvalues

- When $\alpha_z < 4\beta_z$ we get imaginary eigenvalues
- Phase Portrait will spiral in
- Thus we have stability at (0,g)
Stability – Real Eigenvalues

• When $\alpha_z > 4\beta_z$ we get real eigenvalues

• When $\alpha_z > \sqrt{\alpha_z \sqrt{\alpha_z - \beta_z}}$ we get one eigenvalue < 0 while the other is > 0

• Thus our phase portrait is a saddle point that is stable
Stability – Real Eigenvalues

• When $\alpha_z = \sqrt{\alpha_z\alpha_z - \beta_z}$ then one of our eigenvalues equals 0

• This gives us an infinite line of fixed points
Conclusion

- Fixed point (0,g) is both globally attracting and Liapunov stable so we call it asymptotically stable
- Dynamical System always achieves its goal state
- Created this system to help stroke patients