Read your lecture notes and chapters 5 and 6. Submit the following problems. (Topics: Two Dimensional Flows. Linear and nonlinear systems. Phase Portraits.)

1. The matrix \( A = \begin{pmatrix} 4 & -10 \\ 2 & -4 \end{pmatrix} \) has an eigenvalue \( \lambda_1 = 2i \) and a corresponding eigenvector \( v = \begin{pmatrix} 2 + i \\ 1 \end{pmatrix} \).
   
   (a) Find the real solution of the system \( \dot{w} = Aw \).
   
   (b) Each trajectory is an ellipse. Write down the parametric equations of these ellipses: \( x(t) = \ldots \) and \( y(t) = \ldots \).
   
   (c) Along which directions are the major and minor axis of these ellipses are? (Does the eigenvector tell you any information about this?)
   
   (d) Use a graphing utility to plot the coordinates \( x(t) \) and \( y(t) \) as a function of \( t \), when the initial conditions are \( x(0) = -4 \) and \( y(0) = -1 \).
   
   (e) Plot the phase portrait.

2. Analyze the type and stability of the critical points of the following predator-prey ecological system, then plot a phase portrait showing all interactions and the separatrices if any.
   
   \[
   \begin{aligned}
   x'(t) &= 7x - x^2 - 2xy \\
y'(t) &= y - y^2 + 2xy.
   \end{aligned}
   \]

3. Consider the damped nonlinear pendulum
   
   \[
   \theta'' + 0.1\theta' + \sin(\theta) = 0.
   \]
   
   Transform into a first order system, find the critical points, linearize to study the types and the stability of the critical points, then plot a phase portrait, showing the separatrices.

4. Use a phase portrait graphing utility to plot the phase portraits of the systems in problems 1, 2, and 3 above, hence confirming your predictions.

5. 5.1.10 (a, d), 5.1.11 (a,d).

6. 6.2.1.

7. 6.3.10.

8. 6.3.11.