"Risk is a most slippery and elusive concept. It's hard for investors -- let alone economists -- to agree on a precise definition,"

With any financing or investment decision, there is some uncertainty about its outcome. **Uncertainty** is not knowing exactly what will happen in the future. There is uncertainty in most everything we do as financial managers because no one knows precisely what changes will occur in such things as tax laws, consumer demand, the economy, or interest rates. Though the terms "risk" and "uncertainty" are many times used to mean the same, there is a distinction between them. Uncertainty is not knowing what's going to happen.

**Risk** is how we characterize how much uncertainty exists: the greater the uncertainty, the greater the risk. Risk is the degree of uncertainty. In financing and investment decisions there are many types of risk we must consider.

The types of risk a financial manager faces include:

- cash flow risk;
- reinvestment risk;
- interest rate risk; and
- purchasing power risk.

Let's take a look at each of these types of risk.

2. **Cash flow risk**

**Cash flow risk** is the risk that the cash flows of an investment will not materialize as expected. For any investment, the risk that cash flows may not be as expected -- in timing, amount, or both -- is related to the investment's business risk.
A. Business risk

Business risk is the risk associated with operating cash flows. Operating cash flows are not certain because neither are the revenues nor expenditures comprising the cash flows. Regarding revenues: depending on economic conditions and the actions of competitors, prices or quantity of sales (or both) may be different from what is expected. This is sales risk.

Regarding expenditures: operating costs are comprised of fixed costs and variable costs. The greater the fixed component of operating costs, the less easily a company can adjust its operating costs to changes in sales.

The mixture of fixed and variable costs depends largely on the type of business. For example, fixed operating costs make up a large portion of an airline's operating costs: no matter how many passengers are flying, the airline still needs to pay gate fees, pay a pilot, and buy fuel. The variable costs for an airline -- the costs that change depending on the number of passengers -- amount to a little bit of fuel and the cost of the meal (which can't be much!).

Even within the same line of business, companies can vary their fixed and variable costs. For example, an airline could develop a system that allows it to vary the number of cabin stewards and baggage handlers according to passenger traffic, varying more of its operating costs as demand changes. We refer to the risk that comes about from the mix of fixed and variable costs as operating risk. The greater the fixed operating costs, relative to variable operating costs, the greater the operating risk.

Let's take a look at how operating risk affects cash flow risk. Remember back in economics when you learned about elasticity? That's a measure of the sensitivity of changes in one item to changes in another. We can look at how sensitive a firm's operating income are to changes in demand, as measured by unit sales. We'll calculate the operating income elasticity, which we call the degree of operating leverage (DOL).

The degree of operating leverage is the ratio of the percentage change in operating cash flows to the percentage change in units sold. Let's simplify things and assume that we sell all that we produce in the same period. Then,

\[
\text{Degree of operating leverage} = DOL = \frac{\text{percentage change in operating income}}{\text{percentage change in units sold}}
\]

Suppose the price per unit is $30, the variable cost per unit is $20, and the total fixed costs are $5,000. If we go from selling 1,000 units to selling 1,500 units, an increase of 50 percent of the units sold, operating cash flows change from:

<table>
<thead>
<tr>
<th>Item</th>
<th>Selling 1,000 units</th>
<th>Selling 1,500 units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>$30,000</td>
<td>$45,000</td>
</tr>
<tr>
<td>less variable costs</td>
<td>20,000</td>
<td>30,000</td>
</tr>
<tr>
<td>less fixed costs</td>
<td>5,000</td>
<td>5,000</td>
</tr>
<tr>
<td>Operating income</td>
<td>$ 5,000</td>
<td>$10,000</td>
</tr>
</tbody>
</table>

Operating income doubled when units sold increased by 50 percent. What if the number of units decreases by 25 percent, from 1,000 to 750? Operating income declines by 50 percent. What is happening is that for a 1 percent change in units sold, the operating income changes by two
times that percentage, in the same direction. If units sold increased by 10 percent, operating cash flows would increase by 20 percent; if units sold decreased by 10 percent, operating income would decrease by 20 percent.

We can represent the degree of operating leverage in terms of the basic elements of the price per unit, variable cost per unit, number of units sold, and fixed operating costs. Operating income is:

\[
\text{Operating income} = \left[\left(\text{price per unit} \times \text{number of units sold}\right) - \left(\text{variable cost per unit} \times \text{number of units sold}\right)\right] - \text{fixed operating costs}
\]

How much does operating income change when the number of units sold changes? It changes by the difference between the price per unit and the variable cost per unit -- called the *contribution margin* -- times the change in units sold. The percentage change in operating cash flows for a given change in units sold simplifies to:

\[
\text{DOL} = \frac{Q(P-V)}{Q(P-V)-F},
\]

where \( Q \) is the number of units, \( P \) is the price per unit, \( V \) is the variable operating cost per unit, and \( F \) is the fixed operating cost. Therefore, \( P-V \) is the contribution margin per unit.

Applying the formula for DOL using the data in the example, we can figure out the sensitivity to change in units sold from 1,000 units:

\[
\text{DOL @ 1,000 units} = \frac{1,000 ($30-20)}{1,000 ($30-20)-5,000} = \frac{10,000}{5,000} = 2
\]

The DOL of 2.0 means that a 1 percent change in units sold results in a 1 percent x 2.0 = 2 percent change in operating income.

Why do we specify that the DOL is at a particular quantity sold (in this case 1,000 units)? Because the DOL will be different at different numbers of units sold. For example, at 2,000 units,

\[
\text{DOL @ 2,000 units} = \frac{2,000 ($30-20)}{2,000 ($30-20)-5,000} = 1.333
\]

We can see the sensitivity of the DOL for different number of units produced and sold in Exhibit 1. When operating profit is negative, the DOL is negative. At the break-even number of units produced and sold of 500, the DOL is undefined because 500 ($30-20) - $5,000 = $0. The DOL gradually declines when there is a profit as more units are produced and sold.

Let's look at similar situation, but where the firm has shifted some of the operating costs away from fixed costs and into variable costs. Suppose the firm has a unit sales price of $100, a variable cost of $70 a unit, and $10,000 in fixed costs. A change in units sold from 1,000 to 1,500 -- a 50 percent change -- changes operating profit from $20,000 to $35,000, or 75 percent.

The DOL in this case is 1.5:

\[
\text{DOL @ 1,000 units} = \frac{1,000 ($100-70)}{1,000 ($100-70)-10,000} = 1.5
\]
and the change in operating income is 75 percent:

\[
\text{Percentage change in operating income} = \text{DOL} \left( \frac{\text{percentage change in units sold}}{} \right) = 1.5 \times (50\%) = 75\%
\]

But what if, instead, the company is able to reduce fixed costs, shifting some to variable costs? Suppose the fixed costs are now $5,000 and the variable costs are $80 per unit. The DOL at 1,000 units is now 1.33 and the percentage change in operating income is 1.33 (50 percent) = 66.5 percent. We can see the difference in the leverage in these two cases, labeled Case 1 and Case 2, respectively, in Exhibit 2.

Exhibit 2: Profitability and the degree of operating leverage

Case 1: P = $100; V = $70; F = $10,000
Case 2: P = $100; V = $80; F = $5,000

Panel A: Operating income
Panel B: DOL

What we see in this latter example is what we saw a bit earlier in our reasoning of fixed and variable costs: the greater use of fixed, relative to variable operating costs, the more sensitive operating income is to changes in units sold and, therefore, more operating risk.

Both sales risk and operating risk influence a firm's operating cash flow risk. And both sales risk and operating risk are determined in large part by the type of business the firm is in. But management has more opportunity to manage and control operating risk than they do sales risk.

Suppose a firm is deciding on which equipment to buy to produce a particular product. The sales risk is the same no matter what equipment is chosen to produce the product. But the available equipment may differ in terms of fixed and variable operating costs of producing the product. Financial managers need to consider the operating risk associated with their investment decisions.

B. Financial risk

When we refer to the cash flow risk of a security, we expand our concept of cash flow risk. A security represents a claim on the income and assets of a business, therefore the risk of the security is not just the risk of the cash flows of the business, but also the risk related to how these cash flows are distributed among the claimants -- the creditors and owners of the business. Therefore, cash flow risk of a security includes both its business risk and its financial risk.

**Financial risk** is the risk associated with how a company finances its operations. If a company finances with debt, it is a legally obligated to pay the amounts comprising its debts when due. By taking on fixed obligations, such as debt and long-term leases, the firm increases its financial risk. If a company finances its business with equity, either generated from operations (retained earnings) or from issuing new equity,
it does not incur fixed obligations. The more fixed-cost obligations (debt) incurred by the firm, the greater its financial risk.

We can quantify this risk somewhat in the same way we did for operating risk, looking at the sensitivity of the cash flows available to owners when operating cash flows change. This sensitivity, which we refer to as the degree of financial leverage (DFL), is:

$$\text{DFL} = \frac{\text{Percentage change in net income}}{\text{Percentage change in operating income}}$$

Net income is equal to operating income, less interest and taxes. If operating income changes, how does net income change? Suppose operating cash flows change from $5,000 to $6,000 and suppose the interest payments are $1,000 and, for simplicity and wishful thinking, the tax rate is 0 percent:

<table>
<thead>
<tr>
<th>Operating income of $5,000</th>
<th>Operating income of $10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating income</td>
<td>$5,000</td>
</tr>
<tr>
<td>Less interest</td>
<td>1,000</td>
</tr>
<tr>
<td>Income before taxes</td>
<td>$4,000</td>
</tr>
<tr>
<td>Less tax (40 percent)</td>
<td>1,600</td>
</tr>
<tr>
<td>Net income</td>
<td>$2,400</td>
</tr>
</tbody>
</table>

A change in operating income from $5,000 to $6,000, which is a 20 percent increase, increases income before taxes by $1,000 and net income by $600 – each a 25 percent increase.

What if, instead, our fixed financial costs are $3,000? A 20 percent change in operating income results in a 50 percent change in the net income from $1,200 to $1,800. Using more debt financing, which results in more interest expense, increases the sensitivity of owners’ income.

<table>
<thead>
<tr>
<th>Operating income of $5,000</th>
<th>Operating income of $10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating income</td>
<td>$5,000</td>
</tr>
<tr>
<td>less interest</td>
<td>3,000</td>
</tr>
<tr>
<td>Income before taxes</td>
<td>$2,000</td>
</tr>
<tr>
<td>Less tax (40 percent)</td>
<td>800</td>
</tr>
<tr>
<td>Net income</td>
<td>$1,200</td>
</tr>
</tbody>
</table>

We can write the sensitivity of owners' cash flows to a change in operating income, continuing the notation from before and including the fixed financial cost, I, as:

$$\text{DFL} = \frac{[Q(P-V)-F](1-t)}{[Q(P-V)-F-I](1-t)} = \frac{[Q(P-V)-F]}{[Q(P-V)-F-I]}$$

In the case where:

- Number of units sold = Q = 1,000
- Price per unit = P = $30
- Variable cost per unit = V = $20
- Fixed operating costs = F = $5,000
- Fixed financing costs = I = $1,000

$$\text{DFL@1,000 units} = \frac{1,000($30-20) - 5,000}{1,000($30-20) - 5,000 - 1,000} = \frac{5,000}{4,000} = 1.25$$

If fixed financial costs are $3,000, the DFL is equal to 2.5:

*Types of risk*, a reading prepared by Pamela Peterson Drake
Again, we need to qualify our degree of leverage by the level of production since DFL is different at different levels operating income.

The greater the use of financing sources that require fixed obligations, such as interest, the greater the sensitivity of cash flows to owners to changes in operating cash flows.

C. Operating and financial risk

The degree of operating leverage gives us an idea of the sensitivity of operating cash flows to changes in sales. And the degree of financial leverage gives us an idea of the sensitivity of owners' cash flows to changes in operating cash flows. But often we are concerned about the combined effect of both operating leverage and financial leverage. Owners are concerned about the combined effect because both contribute to the risk associated with their future cash flows. And financial managers, making decisions to maximize owners' wealth, need to be concerned with how investment decisions (which affect the operating cost structure) and financing decisions (which affect the capital structure) affect owners' risk.

Let's look back on the example using fixed operating costs of $5,000 and fixed financial costs of $1,000. The sensitivity of owners' cash flow to a given change in units sold is affected by both operating and financial leverage. Consider increasing the units sold up 50 percent. If there was no interest (and therefore no financial leverage), the owners' cash flow would equal operating cash flow. Then a 50 percent increase in units sold would result in a 100 percent increase in cash flows to owners. Now consider decreasing units sold by 50 percent. This would result in a 100 percent decrease in cash flows to owners. But if there is financial leverage, this leverage exaggerates the effect of operating leverage. Consider again the case where there is $1,000 of interest:

<table>
<thead>
<tr>
<th>Units produced and sold</th>
<th>1,000</th>
<th>1,500</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues</td>
<td>$30,000</td>
<td>$45,000</td>
<td>$15,000</td>
</tr>
<tr>
<td>less variable costs</td>
<td>20,000</td>
<td>30,000</td>
<td>10,000</td>
</tr>
<tr>
<td>less fixed costs</td>
<td>5,000</td>
<td>5,000</td>
<td>5,000</td>
</tr>
<tr>
<td>Operating income</td>
<td>$5,000</td>
<td>$10,000</td>
<td>$0</td>
</tr>
<tr>
<td>less interest</td>
<td>1,000</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Income before tax</td>
<td>$4,000</td>
<td>$9,000</td>
<td>-$1,000</td>
</tr>
<tr>
<td>less tax</td>
<td>1,600</td>
<td>3,600</td>
<td>-400</td>
</tr>
<tr>
<td>Net income</td>
<td>$2,400</td>
<td>$5,400</td>
<td>-$600</td>
</tr>
</tbody>
</table>

If units sold increases by 50 percent, from 1,000 to 1,500 units,
- operating income increases from $5,000 to $10,000, or 100 percent, and
- net income increases from $2,400 to $5,400, or 125 percent.

If units sold decrease by 50 percent, from 1,000 to 500 units,
- operating income decreases from $5,000 to $0, or 100 percent, and
- net income decreases from $2,400 to -$600, or 125 percent.

Combining a firm's degree of operating leverage with its degree of financial leverage results in the degree of total leverage (DTL), a measure of the sensitivity of the cash flows to owners to changes in unit sales:

\[
\text{DTL} = \frac{\% \text{ change in net income}}{\% \text{ change in the number of units sold and produced}}
\]
and which is also equal to:

\[
\text{DTL} = \frac{Q(P-V)}{Q(P-V)-F} \times \frac{[Q(P-V)-F](1-t)}{[Q(P-V)-F-I](1-t)} = \frac{Q(P-V)}{Q(P-V)-F-I} = \text{DOL} \times \text{DFL}.
\]

Suppose:

- Number of unit sold \(= Q = 1,000\)
- Price per unit \(= P = $30\)
- Variable cost per unit \(= V = $20\)
- Fixed operating cost \(= F = $5,000\)
- Fixed financing cost \(= I = $1,000\)

Then,

\[
\text{DTL} = \frac{1,000($30-$20)}{1,000($30-$20)-$5,000-$1,000} = 2.5
\]

which we could also have gotten from multiplying the DOL, 2, by the DFL, 1.25. This means that a 1 percent increase in units sold will result in a 2.5 percent increase in net income; a 50 percent increase in units sold results in a 125 percent increase in net income; a 5 percent decline in units sold results in a 12.5 percent decline in income to owners; and so on. We can see the DOL, DFL and DTL for different number of units produced and sold for this example in Exhibit 3.

In the case of operating leverage, the fixed operating costs act as a fulcrum: the greater the proportion of operating costs that are fixed, the more sensitive are operating cash flows to changes in sales. In the case of financial leverage, the fixed financial costs, such as interest, act as a fulcrum: the greater the proportion of financing with fixed cost sources, such as debt, the more sensitive cash flows available to owners are to changes in operating cash flows.

Combining the effects of both types of leverage, we see that fixed operating and financial costs together act as a fulcrum that increases the sensitivity of cash flows available to owners.
Try it! DOL, DFL and DTL

Consider a company that manufacturers and sells a single product, the X product. It costs the company €1 per unit of X in variable costs. Fixed operating costs are €30,000. The company also has fixed financial costs of €10,000. The company expects to sell each unit of X for €5. Complete the following chart:

<table>
<thead>
<tr>
<th>Production</th>
<th>DOL</th>
<th>DFL</th>
<th>DTL</th>
<th>Operating profit</th>
<th>Net profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>15,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solutions are available at the end of this reading.

D. Default risk

When you invest in a bond, you expect interest to be paid (usually semi-annually) and the principal to be paid at the maturity date. But not all interest and principal payments may be made in the amount or on the date expected: interest or principal may be late or the principal may not be paid at all! The more burdened a firm is with debt -- required interest and principal payments -- the more likely it may be unable to make payments promised to bondholders and the more likely there may be nothing left for the owners. We refer to the cash flow risk of a debt security as default risk or credit risk.

Technically, default risk on a debt security depends on the specific obligations comprising the debt. Default may result from:

- failure to make an interest payment when promised (or within a specified period),
- failure to make the principal payment as promised,
- failure to make sinking fund payments (that is, amounts set aside to pay off the obligation), if these payments are required,
- failure to meet any other condition of the loan, or
- bankruptcy.

Why do financial managers need to worry about default risk?

- Because they invest their firm's funds in the debt securities of other firms and they want to know what default risk lurks in those investments;
- Because they are concerned how investors perceive the risk of the debt securities their own firm issues; and

Because the greater the risk of a firm's securities, the greater the firm's cost of financing. We can see this in Exhibit 4, where we show the yields on Aaa and Baa-rated bonds over the period 1919-2005.¹ Aaa-rated bonds have less default risk than Baa-rated bonds, and hence have lower yields. You may notice a wider spread between the Aaa and Baa-rated bond yields during the depression and in recessionary periods.

¹ These yields are for bond ratings by Moody’s Investor Service. The equivalent Standard & Poor's ratings are AAA and BBB.
Default risk is affected by both business risk - which includes sales risk and operating risk -- and financial risk. We need to consider the effects operating and financing decisions have on the default risk of the securities a firm issues, since the risk accepted through the financing decisions affects the firm's cost of financing.

3. Reinvestment rate risk

Another type of risk is the uncertainty associated with reinvesting cash flows, not surprisingly called **reinvestment rate risk**.

Suppose you buy a U.S. Treasury Bond that matures in five years. There is no default risk, since the U.S. government could simply print more money to pay the interest and principal. Does this mean there is no risk when you own a Treasury bond? No. You need to do something with the interest payments as you receive them and the principal amount when it matures. You could stuff them under your mattress, reinvest in another Treasury bond, or invest them otherwise. If yields have been falling, however, you cannot reinvest the interest payments from the bond and get the same return you are getting on the bond.

From the time that you bought the Treasury bond until its maturity five years later, yields on investments may have changed. While the yield on the mattress option hasn't changed (it's still 0 percent), the yields on other investments may have changed. When your Treasury bond matures, you face reinvestment risk.

If we look at an investment that produces cash flows before maturity or sale, such as a stock (with dividends) or a bond (with interest), we face a more complicated reinvestment problem. In this case we're concerned with the reinvestment of the final proceeds (at maturity or sale), but also with the reinvestment of the intermediate dividend or interest cash flows (between purchase and maturity or sale).

Let's look at the case of a five-year bond issued by Company Y, that pays 10 percent interest (at the end of each year, to keep things simple), and has a par value of $1,000. This bond is a coupon bond; that is,
interest is paid at the coupon rate of 10 percent per year, or $100 per bond. If you buy the bond when it is issued at the beginning of 2005 and hold it to maturity, you will have the following cash flows:

### Company One Bond

<table>
<thead>
<tr>
<th>Date</th>
<th>Cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1, 2005</td>
<td>-$1,000.00</td>
</tr>
<tr>
<td>December 31, 2005</td>
<td>100.00</td>
</tr>
<tr>
<td>December 31, 2006</td>
<td>100.00</td>
</tr>
<tr>
<td>December 31, 2007</td>
<td>100.00</td>
</tr>
<tr>
<td>December 31, 2008</td>
<td>100.00</td>
</tr>
<tr>
<td>December 31, 2009</td>
<td>1,100.00</td>
</tr>
</tbody>
</table>

You face five reinvestment decisions along the life of this bond: the four intermediate flows at the end of each year, and the last and largest cash flow that consists of the last interest payment and the par value.

Suppose we wish to compare the investment in the Company One bond with another five-year bond, issued by Company Two, that has a different cash flow stream, but a yield that is nearly the same. Company Two's bond is a zero-coupon bond; that is, it has no interest payments, so the only cash flow to the investor is the face value at maturity:

### Company Two bond

<table>
<thead>
<tr>
<th>Date</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1, 2005</td>
<td>-$1,000.00</td>
</tr>
<tr>
<td>December 31, 2009</td>
<td>+$1,610.51</td>
</tr>
</tbody>
</table>

Both bonds have the same annual yield-to-maturity of 10 percent. If the yield is the same for both bonds, does this mean that they have the same reinvestment rate risk? No. Just from looking at the cash flows from these bonds we see there are intermediate cash flows to reinvest from Company One's bond, but not from Company Two's bond.

Let's see just how sensitive the yield on the investment is to changes in the assumptions on the reinvestment of intermediate cash flows. Suppose we are not able to reinvest the interest payments at 10 percent, but rather at 5 percent per year. We calculate the yield on the bonds assuming reinvestment at 5 percent -- a modified internal rate of return -- by calculating the future value of the reinvested cash flows and determining the discount rate that equates the original investment of $1,000 to this future value:

### Date | Company One bond | Company Two bond |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec. 31, 2005</td>
<td>$100.00</td>
<td>$121.55</td>
</tr>
<tr>
<td>Dec. 31, 2006</td>
<td>100.00</td>
<td>115.76</td>
</tr>
<tr>
<td>Dec. 31, 2007</td>
<td>100.00</td>
<td>110.25</td>
</tr>
<tr>
<td>Dec. 31, 2008</td>
<td>100.00</td>
<td>105.00</td>
</tr>
<tr>
<td>Dec. 31, 2009</td>
<td>1,100.00</td>
<td>1,100.00</td>
</tr>
</tbody>
</table>

Future value with cash flows reinvested 5 percent

$1,552.00 $1,610.50

Using the value of the cash flow as of December 31, 2009 as the future value and the $1,000 investment as the present value, the modified internal rates of return are 9.19 percent for Company One's bond and 10 percent for Company Two's bond. You'll notice that the modified internal rate of return for Company Two's bond is higher than that of Company One's bond, even though the yield to maturity is the same.

2 Is there a short cut in a financial calculator? Not really. Most financial calculators do not have the necessary program for a net future value, so you are left with calculating this type of problem using the time value of money programs to calculate the individual future values and then summing these.
Two's bond is the same as its yield-to-maturity -- because there are no intermediate cash flows. This is shown in Exhibit 5.

What we learn is that if we compare two bonds with the same yield-to-maturity and the same time to maturity, the bond with the greater coupon rate has more reinvestment rate risk. That's because it has more of its value coming sooner in the form of cash flows.

Two types of risk closely related to reinvestment risk of debt securities are prepayment risk and call risk. Consider the case of mortgage-backed securities. These are securities that represent a collection of home mortgages. An intermediary, such as a large finance company, will pool the mortgages together and then sell interests in these mortgages; as the home owner pays interest, this interest is passed on to the investor in this pool of mortgages. In most cases, a home owner is permitted to prepay the mortgage, either in whole or in part; that is, a home owner may pay off the mortgage early. If paid off early, investors in mortgage-backed securities get paid off early, requiring them to reinvest cash flows from this investment earlier than expected. The problem? Home owners tend to refinance when interest rates are low, hence prepaying on the higher rate mortgages. When investors receive these earlier cash flows, they reinvest them at a time of lower interest rates; therefore their investment opportunities are much lower-yielding than the mortgage-backed securities they had invested in.

Investors in securities that can be paid off earlier than maturity face prepayment risk -- the risk that the borrower may choose to prepay the loan -- which causes the investor to have to reinvest the funds.

Call risk is the risk that a callable security will be called by the issuer. If you invest in a callable security, there is a possibility that the issuer may call it in (buy it back). While you may receive a call premium (a specified amount above the par value), you have to reinvest the funds you receive.

There is reinvestment risk for assets other than stocks and bonds, as well. If you are investing in a new product -- investing in assets to manufacture and distribute it -- you expect to generate cash flows in future periods. You face a reinvestment problem with these cash flows: What can you earn by investing these cash flows? What are your future investment opportunities?

**Exhibit 5**  
*The modified internal rate of return for Company One and Company Two bonds for different reinvestment rates*

<table>
<thead>
<tr>
<th>Reinvestment rate</th>
<th>0%</th>
<th>2%</th>
<th>4%</th>
<th>6%</th>
<th>8%</th>
<th>10%</th>
<th>12%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company One bond</td>
<td>8%</td>
<td>10%</td>
<td>12%</td>
<td>12%</td>
<td>10%</td>
<td>8%</td>
<td>6%</td>
</tr>
<tr>
<td>Company Two bond</td>
<td>10%</td>
<td>12%</td>
<td>14%</td>
<td>12%</td>
<td>10%</td>
<td>8%</td>
<td>6%</td>
</tr>
</tbody>
</table>

**Modifying the internal rate**

A bond's yield is the bond's internal rate of return. The internal rate of return assumes that any cash flows from the asset (interest, in the case of a bond) are reinvested at a rate equal to the bond's yield.

A modified internal rate of return is the return on an asset, assuming that intermediate cash flows are reinvested at a specific rate. The modified internal rate of return can provide a more realistic return on an investment since it accounts for reinvestment at some rate other than the asset's internal rate.

Assuming that we can invest intermediate cash flows (e.g., interest) at the current yield until the maturity of the investment is often not realistic.
If we assume that investors do not like risk -- a safe assumption -- then they will want to be compensated if they take on more reinvestment rate risk. The greater the reinvestment rate risk, the greater the expected return demanded by investors.

Reinvestment rate risk is relevant in our investment decisions no matter the asset and we must consider this risk in assessing the attractiveness of investments. The greater the cash flows during the life of an investment, the greater the reinvestment rate risk of the investment. And if an investment has a greater reinvestment rate risk, this must be factored into our decision.

4. Interest rate risk

*Interest rate risk* is the sensitivity of the change in an asset's value to changes in market interest rates. And, you should remember that market interest rates determine the rate we must use to discount a future value to a present value. The value of any investment depends on the rate used to discount its cash flows to the present. If the discount rate changes, the investment's value changes.

Suppose we make an investment in a project that we expect to have in operation for ten years. Two years into the project, we look at our investment opportunities and see that the returns on alternative investments have increased. Does this affect our value of this two-year-old project? Sure: we now have a higher opportunity cost - the return on our best investment opportunity -- and therefore the value of the two-year-old project is now less and we need to assess whether to continue or terminate the project.

And this works if the opportunity cost declines as well. If the return on our next best investment opportunity declines, the existing project will look even better. Interest rate risk also is present in debt securities. If you buy a bond and intend to hold it until its maturity, you don't need to worry about its value changing as interest rates change: your return is the bond's yield-to-maturity. But if you do not intend to hold the bond to maturity, you need to worry about how changes in interest rates affect the value of your investment. As interest rates go up, the value of your bond goes down. As interest rates go down, the value of your bond goes up.

Let's compare the change in the value of the Company One bond to the change in the value of the Company Two bond as the market interest rate changes. Suppose that it is now January 1, 2006; that is, one year after the bonds were issued.

- If yields remain at 10 percent, the value of the bonds are $1,000 and $1,099.99 for the Company One and Company Two bonds, respectively.
- If market interest rates change causing the bonds to yield 12 percent, the value of the Company One and Company Two bonds, respectively, are $939.98 and $1,060.89.

But how sensitive are the values of the bond to changes in market interest rates? If the bonds' yield changed on January 1, 2001 from 10 percent to 12 percent, the value of Company One bond would drop from $1,000.00 to $938.26 -- a drop of $61.74, or 6.17 percent of the bond's value. The drop would be greater for Company Two's bond -- a drop of $76.50 or 6.95 percent of its value. Looking at changes in the value of the bonds for different yield changes, we see that the Company Two bond's value is more sensitive to changes in yields than is Company One's:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Yield change from 10% to 12%</th>
<th>Yield change from 10% to 14%</th>
<th>Yield change from 10% to 8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company One</td>
<td>-$61.74 or -6.17%</td>
<td>-$116.54 or -11.65%</td>
<td>+$66.23 or +6.62%</td>
</tr>
<tr>
<td>Company Two</td>
<td>-$76.50 or -6.95%</td>
<td>-$146.45 or -13.31%</td>
<td>+$83.77 or +7.62%</td>
</tr>
</tbody>
</table>

We can see this graphically in Exhibit 5.
We can make some generalizations about the sensitivity of a bond's value to changes in yields.

- For a given coupon rate, the longer the maturity of the bond, the more sensitive the bond's value to changes in market interest rates. Why? Because more of the bond's value is farther out into the future (the principal payments) and the more the present value is affected by a change in the discount rate.

- For a given maturity, the lower the coupon rate, the more sensitive the bond's value to a change in the yield. Why? The greater the coupon rate, the more of the bond's present value is derived from cash flows that are affected less by discounting.

Example: Maturity and interest rate risk

Compare the change in the value of two bonds that have the same coupon rate, 10 percent and the same face value, $1,000, with interest paid annually. If Bond SM has five years remaining to maturity and Bond LM has ten years remaining to maturity, a change in the yield on the bonds from 10 percent to 12 percent results in a greater change in Bond LM's value.

When interest changes from 10 percent to 12 percent,

- Bond SM's value changes from $1,000 to $928, down 7.2 percent
- Bond LM's value changes from $1,000 to $887, down 11.3 percent

Example: Coupon rate and interest rate risk

Compare two bonds that have the same time remaining to maturity, five years, the same face value, $1,000, and both are priced to yield 10 percent. If Bond HC has a 10 percent coupon and Bond LC has a 5 percent coupon, a change in the yield has a greater effect of the value of Bond LC than on Bond HC.

When yields change from 10 percent to 12 percent,

- Bond HC's value changes from $1,000 to $928, down 7.20 percent
- Bond LC's value changes from $810 to $748, down 7.65 percent
Try it! Interest rate sensitivity

Consider two bonds: Bond ABC and Bond XYZ. Bond ABC has a coupon rate of 5 percent and matures in six years. Bond XYZ has a coupon rate of 8 percent and matures in two years. Calculate the value of both of these bonds for the different yields to maturity, completing this table:

<table>
<thead>
<tr>
<th>Yield to maturity</th>
<th>Bond ABC</th>
<th>Bond XYZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 percent</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Which bond's value is more sensitive to changes in the yield to maturity?

5. Purchasing power risk

**Purchasing power risk** is the risk that the price-level may increase unexpectedly. If a firm locks in a price on your supply of raw materials through a long-term contract and the price-level increases, it benefits from the change in the price level and your supplier loses - the firm pays the supplier in cheaper currency. If a firm borrows funds by issuing a long-term bond with a fixed coupon rate and the price-level increases, the firm benefits from an increase in the price level and its creditor is harmed since interest and the principal are repaid in a cheaper currency.

Consider the 4 percent and 6 percent inflation rates for the years 20X1 and 20X2, respectively. If you borrowed $1,000 at the beginning of 20X1 and paid it back two years later, you are paying back $1,000 in end-of-20X2 dollars. But how much is a 20X2 dollar worth relative to beginning-of-20X1 dollars? We can use the compounding relation to work this out. We know that the future value is $1,000. We also know that the rate of inflation over the two year period is determined from compounding the two inflation rates:

\[ r = (1 + \text{inflation rate for 20X1})(1 + \text{inflation rate for 20X2}) - 1 \]

\[ r = (1 + 0.04)(1 + 0.06) - 1.0000 = 1.1024 - 1.0000 = 10.24\text{ percent over 20X1 and 20X2} \]

We can solve the basic valuation relation for today's value, PV, considering r to be a two-year rate (that is, a period is defined as the two-year stretch from the beginning of 20X1 through the end of 20X2):

\[ \text{FV} = \text{PV} (1 + r) \]

\[ $1,000 = \text{PV} (1 + 0.1024), \]

and rearranging to solve for PV,

\[ \text{PV} = \frac{$1,000}{1.1024} = $907.11 \]

Therefore, the $1,000 you paid back at the end of 20X2 was really only worth $907.11 at the beginning of 20X1. As a borrower, you have benefited from inflation and your lender has lost.

How much should your lender have demanded just to keep up with inflation? That is, how much should your lender have demanded without any compensation for the time-value-of-money or for the uncertainty that you will pay it back?

\[ \text{FV} = $1,000 (1 + 0.1024) = $1,102.40. \]

Demanding $1,102.40 in return at the end of 20X2 would have *just* compensated your lender for the purchasing power loss since the beginning of 20X1.
The extent to which inflation is anticipated is reflected in interest rates. Let's refer to the return after considering inflation as the **real return** and refer to the return before removing inflation as the **nominal return**. Therefore,

\[
\text{Nominal return} + 1 = \left(1 + \frac{\text{Inflation rate}}{\text{Real return}}\right) \left(1 + \text{Real return}\right).
\]

This relation between the nominal return, the inflation rate, and the real return is referred to as the **Fisher effect**.

If we solve for the nominal return, we can state this return in terms of the inflation rate, the real rate, and the cross-product:

\[
\text{Nominal return} = \left[\left(1 + \frac{\text{Inflation rate}}{\text{Real return}}\right) \left(1 + \text{Real return}\right)\right] - 1
\]

As you can see, the nominal return is comprised of three parts: the inflation rate, the real return, and the cross-product of the inflation rate and the real return. Because this cross-product term is usually quite small -- 0.24 percent or 0.0024 in the last example -- we often leave it out and consider the nominal return to be the sum of the inflation rate and the real return.

\[
\text{Nominal return} = \text{Inflation rate} + \text{real return}
\]

The difference between the nominal return and the real return is often referred to as the **inflation premium**, because it is the additional return necessary to compensate for inflation.

Anticipated inflation is incorporated into interest rates and valuations. The risk associated with purchasing power is the risk that there will be unanticipated changes in inflation that will affect the purchasing power. Any unanticipated changes in inflation will affect the parties on both sides of the transaction, with one party gaining as the other losing.

You can see the impact of inflation on interest rates in Exhibit 7, where the real and nominal rates of interest are graphed from January 2003 through January 2006. The widening of the span between these two rates in 2004 and 2005 raised concerns for inflation, prompting the Federal Reserve to raise the Federal Funds rate.
6. Returns and the tolerance for bearing risk

Which product investment do you prefer, A or B? Most people would choose A since it provides the same expected return, with less risk. Most people do not like risk -- they are risk averse. Risk aversion is the dislike for risk. Does this mean a risk averse person will not take on risk? No -- they will take on risk if they feel they are compensated for it.

A risk neutral person is indifferent towards risk. Risk neutral persons do not need compensation for bearing risk. A risk preferent person likes risk -- someone even willing to pay to take on risk. Are there such people? Yes. Consider people who play the state lotteries, where the expected value is always negative: the expected value of the winnings is less than the cost of the lottery ticket.

When we consider financing and investment decisions, we assume that most people are risk averse. Managers, as agents for the owners, make decisions that consider risk “bad” and that if risk must be borne, they make sure there is sufficient compensation for it.

Example: Risk aversion and investor choices

Risk averse investors prefer more return to less, and prefer less risk to more.

Consider the following investments and the associated expected return and risk (measured by standard deviation):

<table>
<thead>
<tr>
<th>Investment</th>
<th>Expected return</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10%</td>
<td>12%</td>
</tr>
<tr>
<td>B</td>
<td>10%</td>
<td>11%</td>
</tr>
<tr>
<td>C</td>
<td>11%</td>
<td>12%</td>
</tr>
<tr>
<td>D</td>
<td>11%</td>
<td>11%</td>
</tr>
<tr>
<td>E</td>
<td>9%</td>
<td>10%</td>
</tr>
<tr>
<td>F</td>
<td>12%</td>
<td>13%</td>
</tr>
</tbody>
</table>

If you are a risk-averse investor, which investment would you prefer of each of the following pairs:

- A or B? A or C?
- C or D? A or D?
- D or E? D or F?
- E or F? C or F?

Some choices are clear, and some are not. Some, like the choice between D and E, depend on the investor’s individual preferences for risk and return tradeoff, which we refer to as their utility function.
bearing it. As agents for the owners, managers cannot have the “fun” of taking on risk for the pleasure of doing so.

Risk aversion is the link between return and risk. To evaluate a return you must consider its risk: Is there sufficient compensation (in the form of an expected return) for the investment's risk?

7. Summary

There are many different types of risks that a financial manager faces. We've illustrated a few in this reading, but there are many more risks, some of which are quantifiable, some of which are not. Any investment involves risks and the challenge is to understand what these risks are and how large they are. Because investment decision-making requires a trade-off between benefits and costs, understanding the risk of an investment is essential to understanding the benefits and costs.

8. Solutions to Try it!

**DOL, DFL and DTL**

<table>
<thead>
<tr>
<th>Production</th>
<th>DOL</th>
<th>DFL</th>
<th>DTL</th>
<th>Operating profit</th>
<th>Net profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>15,000</td>
<td>2.000</td>
<td>1.500</td>
<td>3.000</td>
<td>€30,000</td>
<td>€20,000</td>
</tr>
<tr>
<td>20,000</td>
<td>1.600</td>
<td>1.250</td>
<td>2.000</td>
<td>€50,000</td>
<td>€40,000</td>
</tr>
<tr>
<td>30,000</td>
<td>1.333</td>
<td>1.125</td>
<td>1.500</td>
<td>€90,000</td>
<td>€80,000</td>
</tr>
<tr>
<td>40,000</td>
<td>1.231</td>
<td>1.083</td>
<td>1.333</td>
<td>€130,000</td>
<td>€120,000</td>
</tr>
</tbody>
</table>

**Interest rate sensitivity**

<table>
<thead>
<tr>
<th>Yield to maturity</th>
<th>Bond ABC</th>
<th>Bond XYZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 percent</td>
<td>$1,052.88</td>
<td>$1,076.15</td>
</tr>
<tr>
<td>5 percent</td>
<td>$1,000.00</td>
<td>$1,056.43</td>
</tr>
<tr>
<td>6 percent</td>
<td>$950.23</td>
<td>$1,037.17</td>
</tr>
<tr>
<td>7 percent</td>
<td>$903.37</td>
<td>$1,018.37</td>
</tr>
<tr>
<td>8 percent</td>
<td>$859.22</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>9 percent</td>
<td>$817.63</td>
<td>$982.06</td>
</tr>
</tbody>
</table>

In terms of interest rate sensitivity, the bond with the longer maturity and lower coupon has more interest rate sensitivity. We can confirm this by comparing the price changes in the bonds as yields change:

<table>
<thead>
<tr>
<th>Change in yield to maturity</th>
<th>% change in value of Bond ABC</th>
<th>% change in value of Bond XYZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 → 5 percent</td>
<td>-5.02%</td>
<td>-1.83%</td>
</tr>
<tr>
<td>5 → 6 percent</td>
<td>-4.98%</td>
<td>-1.82%</td>
</tr>
<tr>
<td>6 → 7 percent</td>
<td>-4.93%</td>
<td>-1.81%</td>
</tr>
<tr>
<td>7 → 8 percent</td>
<td>-4.89%</td>
<td>-1.80%</td>
</tr>
<tr>
<td>8 → 9 percent</td>
<td>-4.84%</td>
<td>-1.79%</td>
</tr>
</tbody>
</table>