In the addition problem below, each letter stands for a different digit. However, each letter stands for the same digit in every place where it appears. Find the only possible value for each digit to make a correct addition statement:

\[
\begin{array}{cccc}
S & E & N & D \\
+ & M & O & R & E \\
\hline
M & O & N & E & Y \\
\end{array}
\]

The first observation is simply that we must have \( M = 1 \). (Note that we don’t allow 0 to be the first digit of a number.) Two four-digit numbers cannot sum to something greater than 20,000. So, right off the bat, we have this:

\[
\begin{array}{cccc}
S & E & N & D \\
+ & 1 & O & R & E \\
\hline
1 & O & N & E & Y \\
\end{array}
\]

The next observation is that either \( S = 9 \) or \( S = 8 \) with a carry of one from the previous column to make a sum greater than ten. In either case, we see that \( O = 0 \). Our problem now looks like this:

\[
\begin{array}{cccc}
S & E & N & D \\
+ & 1 & 0 & R & E \\
\hline
1 & 0 & N & E & Y \\
\end{array}
\]

Now it is clear that there cannot be a carry from the third to the fourth column, meaning that \( S = 9 \):

\[
\begin{array}{cccc}
9 & E & N & D \\
+ & 1 & 0 & R & E \\
\hline
1 & 0 & N & E & Y \\
\end{array}
\]

Since we cannot have that \( E = N \), we must have a carry from the second column to the third. It follows that \( N = E + 1 \).

This is where things get tricky. Let’s suppose there is no carry from the first column to the second. In this case, we would have \( N + R = 10 + E \), since we need a carry into the third column.
Substituting for $N$ in this equation gives $(E + 1) + R = 10 + E$. This implies that $R = 9$, which is impossible since we have already determined that $S = 9$.

Thus, we must have a carry from the first column into the second. That means that $N + R + 1 = 10 + E$. Once more making our substitution gives us $(E + 1) + R + 1 = 10 + E$, which immediately gets us that $R = 8$. Our problem now looks like this:

$$
\begin{array}{c}
9 & E & N & D \\
+ & 1 & 0 & 8 & E \\
\hline
1 & 0 & N & E & Y \\
\end{array}
$$

Now, since we must have a carry from the first column into the second, we see that $D + E = 10 + Y$. Since 0 and 1 are already taken, it must be that $D + E$ is at least 12. Given the digits that remain available, the only possibilities are that $D$ and $E$ are 5 and 7, or 6 and 7. In either case, one of them is 7. We certainly cannot have that $E = 7$, since then $N = 8$, which is impossible since 8 is already taken. It follows that $D = 7$.

Now we see we cannot have $E = 6$, since then $N = 7$, which is impossible since we know $D = 7$. The only way out is to suppose that $E = 5$ and $D = 7$. The rest of the letters now fall immediately, and we have the solution:

$$
\begin{array}{c}
9 & 5 & 6 & 7 \\
+ & 1 & 0 & 8 & 5 \\
\hline
1 & 0 & 6 & 5 & 2 \\
\end{array}
$$