Math 321, Spring 2010: Examples of two factor studies with different designs.

1. Stain Removal Experiment (Source: Design and Analysis of Experiment, Dean & Voss).

Objectives: Determine if amount of bleach and type of stain affect amount of time for stain removal and whether the effect of one depends on the other.

Treatment Structure:

Factor A: Bleach Concentration (3,5,7 teaspoonfuls of bleach per cup of water)
Factor B: Type of Stain (blue ink, jam, tomato sauce).

9 "treatments" (combinations of bleach concentration and type of stain), 5 replications per treatment combination

Response Variable: Amount of Time for Stain Removal (seconds), total of 45 = 9 x 5 observations on response

Design 1: Completely Randomized Design: choose treatment combination at random, treat piece of cloth with selected type of stain and bleach concentration, measure amount of time for stain removal – repeat another 44 times.

EU = piece of cloth (in a cup of water at a time slot)

Extraneous variables: variation in water from cup to cup, slight variations in amount of stain applied to piece of cloth, slight variations in amount of bleach concentration applied to water, any time effects, differences in pieces of cloths, measurement error, etc.

Randomization: Select a piece of cloth at random. Select a treatment combination at random to assign to this piece of cloth (write down 9 treatment combinations, each 5 times on slips of paper, for a total of 45 slips of paper. Mix thoroughly, pull one out for each test). Note – soaking one piece of cloth at a time.

Design 2: **Split Plot Design:** Choose a type of stain at random, prepare 3 pieces of cloth all with that type of stain. Assign 3 pieces with particular stain to three concentrations in three cups. Measure amount of time for three pieces with particular stain. Repeat by choosing type of stain at random again, assigning 3 pieces of cloth to three concentrations. Keep doing until each stain selected 5 times for a total of 45 observations on amount of time for stain removal.

Soaking three pieces of cloth with same stain type in consecutive time slots.

Randomization: Write down three stain types on pieces of paper, each 5 times, for a total of 15 pieces of paper. Pull slip of paper at random – which ever stain is selected determines the stain type for three pieces of cloth. Select three pieces of cloth at random from lot of pieces to determine which go to what cup. Write down bleach concentrations on three slips of paper, one for each concentration. Select from these three slips to determine which of three cups get which concentrations.

Split Plot Designs have two types of experimental units (whole units and split units.

EU for Type of Stain (whole unit): time period/3 testings
EU for Bleach Concentration (split unit): piece of cloth in cup of water

Whole units serve as blocks in split plot design, here a block is a time period/set of 3 cups of water

Extraneous variables: Same as before, time effects from set of 3 testings to set of 3 testings.

Blocking: In split plot designs, the whole units serve as blocks, here time period when a set of three cups are tested. Blocking controls variation in response due to different time periods when sets of 3 cups are tested.

2. **Paper Towel Absorption Experiment** (Source: Math 321 experiment)

Objectives: Determine if brand of paper towel and type of liquid affect amount of liquid absorbed by paper towel and whether the effect of one factor upon the level of the other factor.

Treatment Strucutre:

Factor A: Brand of Paper Towel (Coronet, Kleenex, Scott)
Factor B: Liquid (Water, Dishwashing Detergent, Vegetable Oil)

9 treatments (combinations of brand and liquid), 5 replications per treatment

Response Variable = amount of liquid (ml) absorbed

Design 1: Completely Randomized Design: At a particular one of the 45 time slots, the brand and liquid to be used are randomly selected. Fifty milliliters of the liquid is poured/measured into a graduated cylinder and then poured into a container. A section of the paper towel is submerged in the container. After 1 minute passes, the paper towel is removed, letting the excess liquid drip off the towel for 30 seconds. The remaining liquid in the container is then poured back in

to the graduated cylinder. The remaining amount is then subtracted from 50 to get the amount of liquid absorbed. This is done 45 times in random order.

EU = soaking of a paper towel in a liquid at a particular time slot

Randomization: Write down 9 treatment combinations, each 5 times on slips of paper, for a total of 45 slips of paper. Mix thoroughly, pull one out for each test.

Extraneous variables: variation in size of sheets, variations in amount of time soaking, measurement error, etc.

Design 2: Randomized Complete Block Design: Suppose because of time constraints can only do one replication of entire experiment in a day (9 trials). Require 5 days. On each day write down the 9 treatment combinations on 9 slips of paper. Pull one out to determine treatment combination for 1^{st} trial, 2^{nd} trial, …, 9^{th} trial. Repeat this on 4 other days.

EU = soaking of one paper towel in a liquid at a particular time slot

Randomization: On a given day select the nine treatments combinations at random to decide which goes first, second, etc. Individual sheets selected at random from roll, etc.

Extraneous variables: Same as before, time effects from day to day.

Block: Set of 9 soakings at 9 time slots on a day, blocking controls day to day variation.

3. Effect of Type of Container on Cooling Time of Coffee and Hot Chocoloate
(Source: Math 321 experiment)

Objective: Determine whether type of liquid and type of container affect how long it takes for the liquid to cool and whether the effect of one factor depends upon the other factor.

Treatment Structure:

Factor A: Beverage Type (Coffee, Hot Chocolate)
Factor B: Container Type (Ceramic mug, Styrofoam cup, paper cup)

6 treatments (combinations of beverage type and container type), 5 replications per treatment

Response Variable = amount of time (seconds) for beverage to cool down to 72

degrees centigrade from starting temperature of 82 degrees centigrade, total of 30 observations (6 x 5).

Design 1: Split Plot Design: Test the 3 different types of containers at the same time using a particular beverage. Type of beverage is selected at random. Enough of this beverage for 3 cups is heated to 82 degrees centigrade. After heating the beverage is split into thirds by pouring into 3 container, one of each type. Amount of time for particular beverage to cool to 72 degrees centigrade is measured.

Split Plot Design has two types of experimental units (whole units and split units).

EU for Type of Beverage (whole unit): heating/cooling portion of beverage at a time slot

EU for Container Type (split unit): 3 smaller portions (thirds) of large portion of beverage. These thirds are called split units because they are a splitting of the larger portion of the beverage.

Extraneous variables: variation in beverage, variation in pouring rate, measurement error, etc.

Blocking: Whole units serve as blocks, here a block is the set of three smaller quantities of beverage tested at a time slot, blocking controls any time effects across the testings.

Randomization: Need to choose a beverage at random (write down on 10 slips of paper the type of beverage, each 5 times). Pull one slip of paper to determine which beverage to use. Heat beverage to desired temperature. Choose one ceramic mug at random from 10, one paper cup at random from 10, one Styrofoam cup at random from 10. Assign at random which third of liquid (first, second, or third) goes to which type of cup.

Design 2: Completely Randomized Design: Test one type of cup and liquid at a time. Randomly select type of liquid to heat and then randomly select type of cup in which to pour liquid. Repeat 29 other times.

EU: heating/cooling of a portion of beverage at a time slot

Randomization: Need to randomly select which combination of beverage and type of container to use. Once type of container is selected need to randomly select one of that type.

Extraneous variables: Same as before.
Blocking: None

4. Dessert experiment (Source: Design and Analysis, Dean & Voss)

Objective: Determine whether the melting rate of a frozen orange dessert would be affected by the addition of salt and sugar and whether the effect of one factor depends upon the effect of the other factor.

Treatment Structure:

Factor A: Amount of Salt added to 12-ounce can of frozen orange juice with 3 cups of water (1/8 tsp salt, ¼ tsp salt)

Factor B: Amount of sugar added to 12-ounce can of frozen orange juice with 3 cups of water (1/4 cup, ½ cup, ¾ cup)

6 treatments (combinations of salt and sugar), 5 replications per treatment

Response Variable: Percentage melting (by weight)

Procedure: The orange juice mixes were frozen in ice cube trays and allocated to random positions in a freezer. After 48 hours, the cubes were removed from the freezer, placed on half-inch mesh wire grid and allowed to melt into a container in a laboratory (which was held at 24.4 degrees Centigrade). The percentage melting (by weight) of the cubes were recorded.

Design 1: Completely Randomized Design: The treatments were assigned completely at random to the 30 12-ounce cans. The mixtures were assigned at random to positions in the freezer.

EU: 12-ounce can of frozen orange juice

Extraneous variables: Variation in orange juice from can to can, variation in water added from can to can, variation in weight of the cubes from tray to tray, variation in microclimate from melting area to melting area.

Randomization: Write down 6 treatment combinations on 30 slips of paper, each treatment combination on 5 slips. Pull one slip, add particular amount of salt and sugar to can with 3 cups of water.

Design 2: Randomized Complete **Block Design.** Suppose that resources (small freezer, etc.) allowed only testing of 6 cans or one replication of all treatments at a time. Use 5 different days to do experiment. On each day, choose 6 cans at random from those to be used. Assign 6 cans to the 6 different different combinations. Test as before. Repeat on 5 other days.

EU: 12-ounce can of frozen orange juice

Block: Set of 6 cans tested on a particular day, blocking controls any effects of day.

Extraneous variables: Same as for completely randomized design.

5. Baking Cupcakes Experiment (Source: Analysis of Messy Data, Milliken & Johnson)

Objectives: Determine the effects of three different recipes and two cooking temperatures on the volume of cupcakes and whether the effect of a recipe depends on the temperature used (or vice-versa).

Treatment Structure:

Factor A: Recipe (3 different recipes: R1, R2, R3)

Factor B: Temperature (2 different temperatures, T1, T2)

6 treatments (combinations of temperature and recipe), 5 replications per treatment combination, thus a total of 30 cupcakes need to be baked.

Response Variable = volume of cupcake

Design 1: Completely Randomized Design: Select at random the treatment combination to use. Mix enough of a batch of the selected recipe to make one cupcake, fill the cupcake form, and then bake that cupcake at the selected temperature. Repeat for the other 29 cupcakes to be made.

EU: batch of dough in the oven at a particular time slot

Extraneous variables: variations in dough, variations in the temperature within the oven, measurement error, etc.

Randomization: Write down on slips of paper the combinations of recipe and temperature, each written down on 5 slips of paper, for a total of 30 clips of paper. Pull one out to determine particular recipe and temperature used.

Design 2: Randomized Complete Block Design. Only six cupcakes can be made in a day. On a given day one replication of the experiment is conducted, that is, 6 cupcakes will be baked, one for each of the 6 combinations of recipe and temperature. As before only one cupcake is baked at a time with the particular recipe and temperature selected at random from the six.

EU: batch of dough in the oven at a time slot

Extraneous Variables: same as for completely randomized design

Block: Set of 6 cupcake bakes in a day, blocking controls any variation associated with days.

Randomization: Write down the 6 treatment combinations of recipe and temperature on 6 slips of paper. On a given day, select from these slips to decide which combination to use.

Design 3: Split Plot Design. Make three cupcakes, one with each of the three recipes. Assume that there is only one oven. Choose an oven temperature at random. Put all three cupcakes in the oven at the selected temperature. Repeat this procedure, 5 times for each of the oven temperatures.

EUs for temperature (whole units): set of three batches in the oven at a time slot

EUs for recipe (split units): the individual batches of dough in the oven

Extraneous variables: Same as before.

Block: In split plot design, the whole units serve as blocks of the split units.

Randomization: Need to randomly select the temperature to use at a particular baking. Need to randomly assign the three cupcakes made with the different recipes to different positions in oven.