Generating Fractals Using Complex Functions

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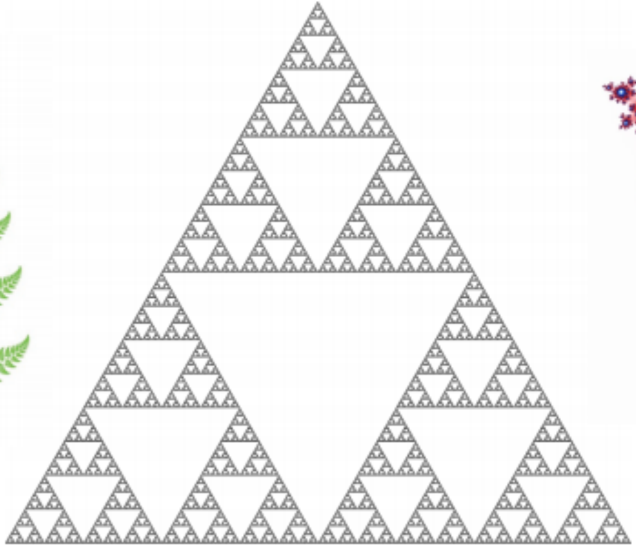
What is a Fractal?

- Fractals are infinitely complex patterns that are self-similar across different scales.
- Created by repeating simple processes over and over in a feedback loop.
- Often represented on the complex plane as 2-dimensional images

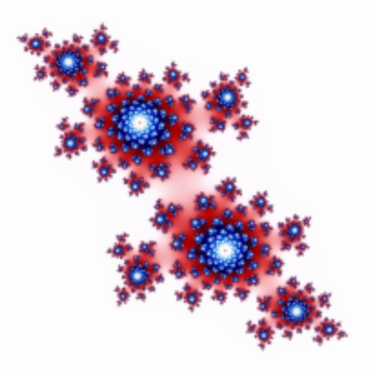
Where do we find fractals?

In Nature

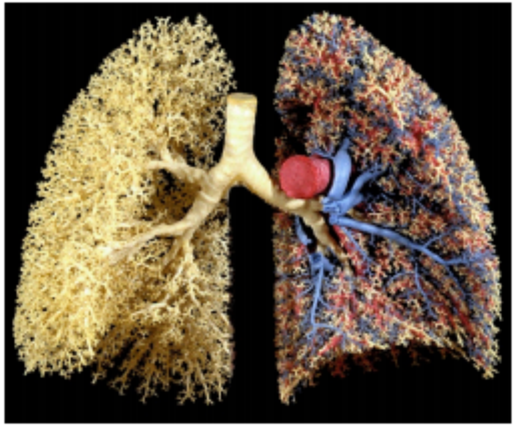
In Geometry



In Algebra

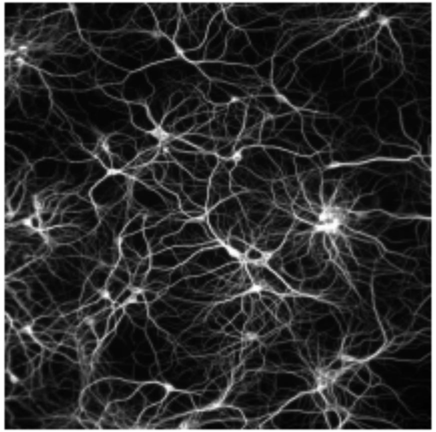


Fractals in Nature



Lungs

Oak Tree

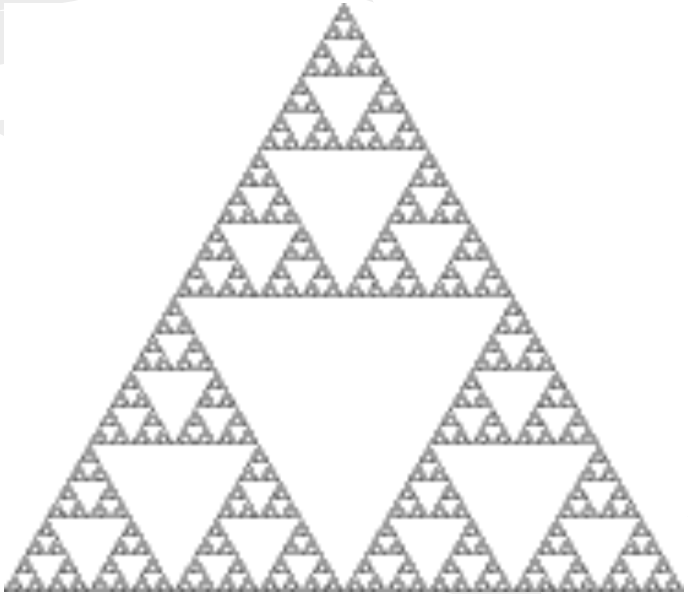


Neurons from the human cortex

Regardless of scale, these patterns are all formed by repeating a simple branching process.

Geometric Fractals

"A rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole." Mandelbrot (1983)



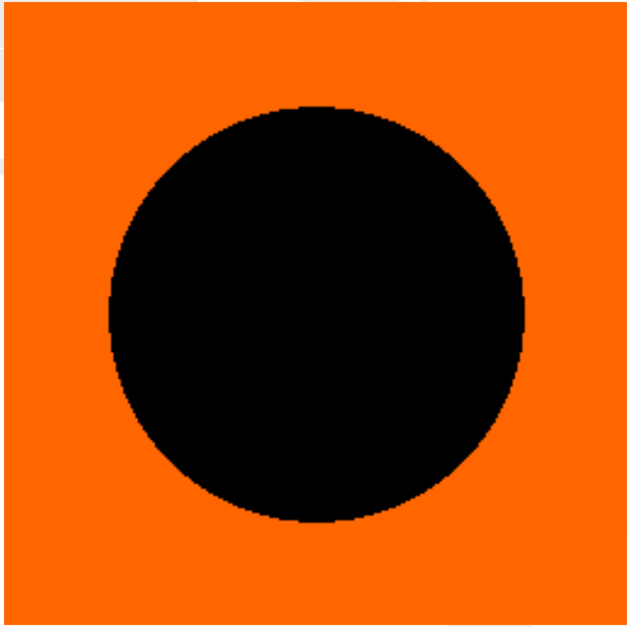
The Sierpinski Triangle

Algebraic Fractals

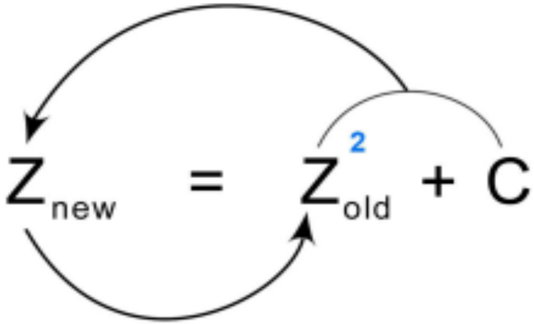
- Fractals created by repeatedly calculating a simple equation over and over.
- Were discovered later because computers were needed to explore them
- Examples:
  - Mandelbrot Set
  - Julia Set
  - Burning Ship Fractal

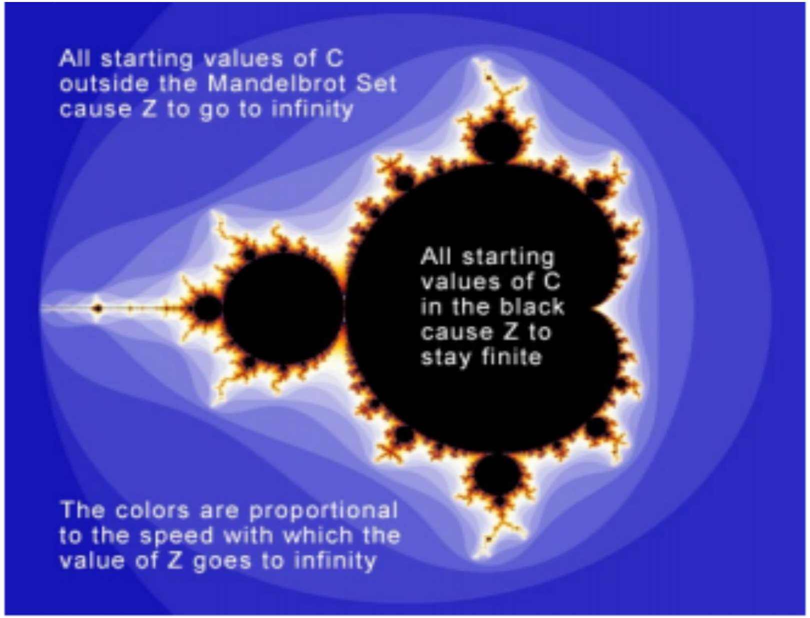
Mandelbrot Set

- Benoit Mandelbrot discovered this set in 1980, shortly after the invention of the personal computer
- $z_{n+1} = z_n^2 + c$
- That is, a complex number *c* is part of the Mandelbrot set if, when starting with $z_0 = 0$ and applying the iteration repeatedly, the absolute value of z_n remains bounded however large *n* gets.



Animation based on a static number of iterations per pixel.

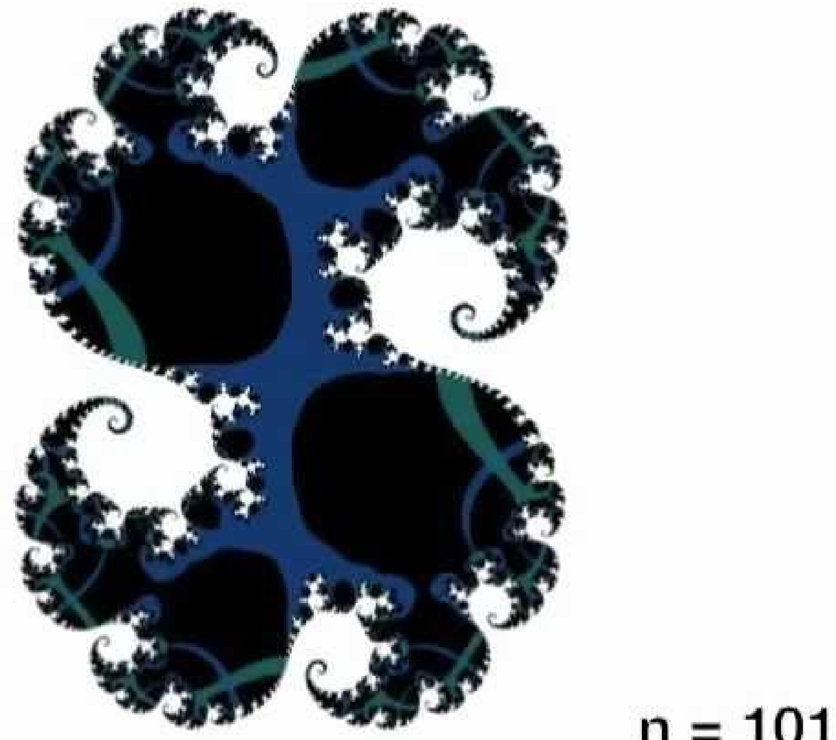




The Mandelbrot set is the complex numbers c for which the sequence (c, $c^2 + c$, $(c^2+c)^2 + c$, $((c^2+c)^2+c)^2 + c$, $(((c^2+c)^2+c)^2+c)^2 + c$, ...) does not approach infinity.

Julia Set

- Closely related to the Mandelbrot fractal
- Complementary to the Fatou Set

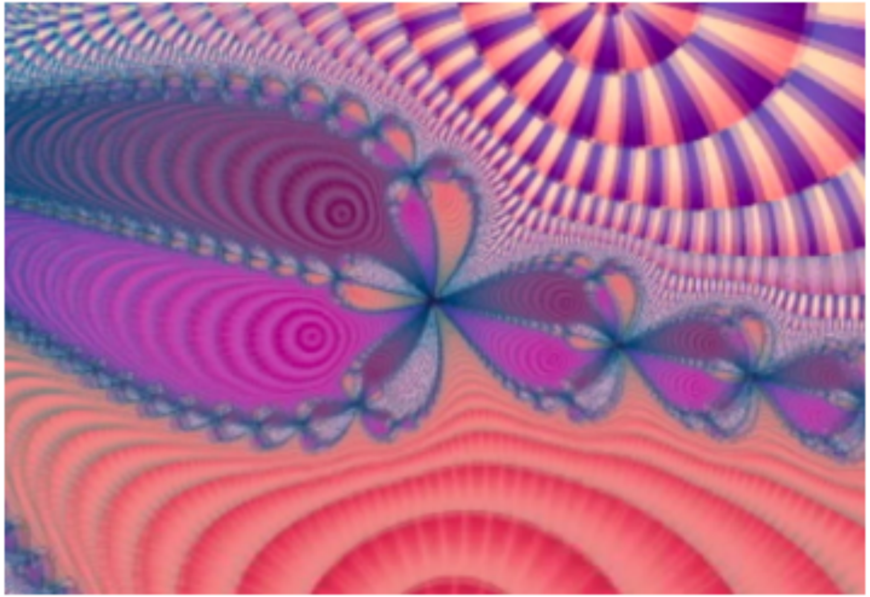


Featherino Fractal

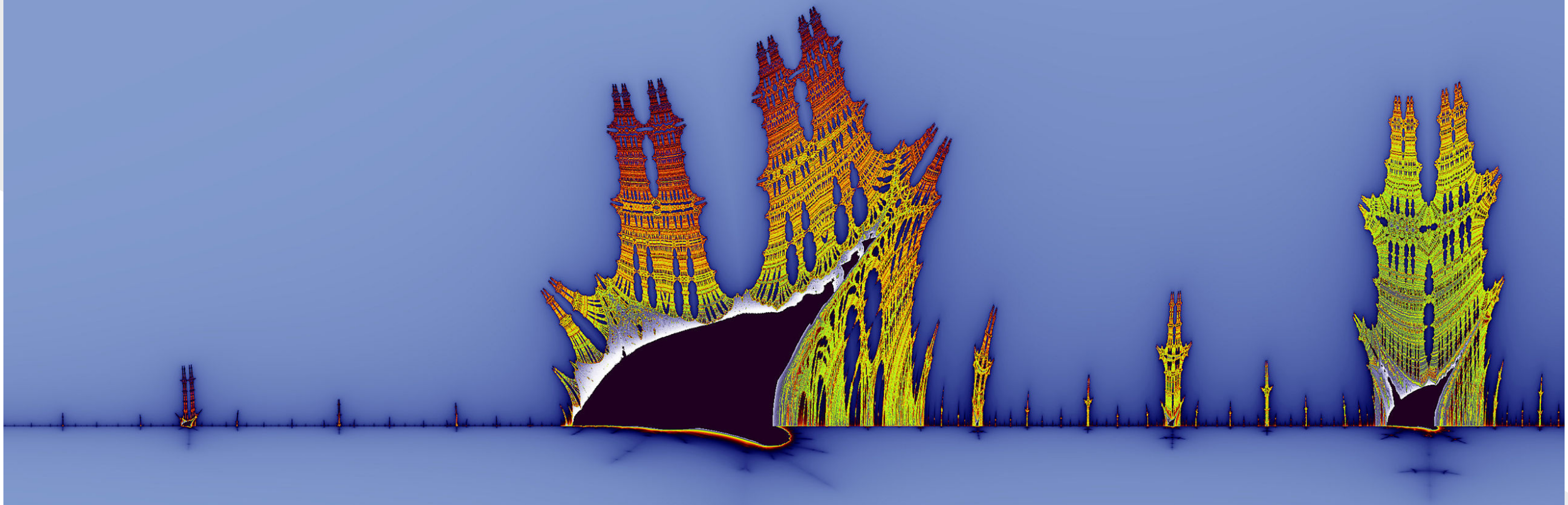
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Newton’s method for the roots of a real valued function

$$Z_{new} = Z_{old} - a\frac{(Z_{old}^4 - 1)}{4Z_{old}^3}$$

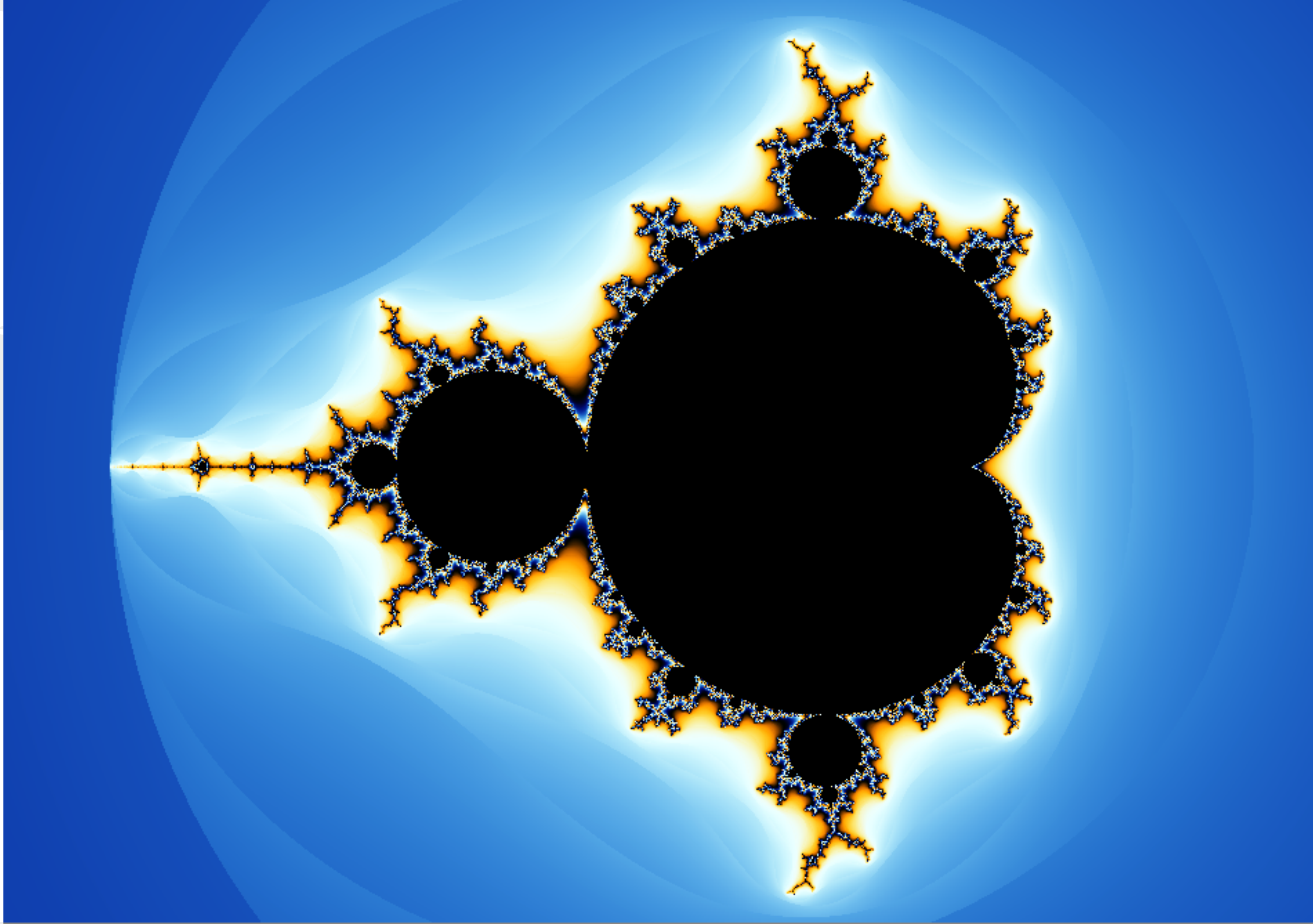


Burning Ship Fractal

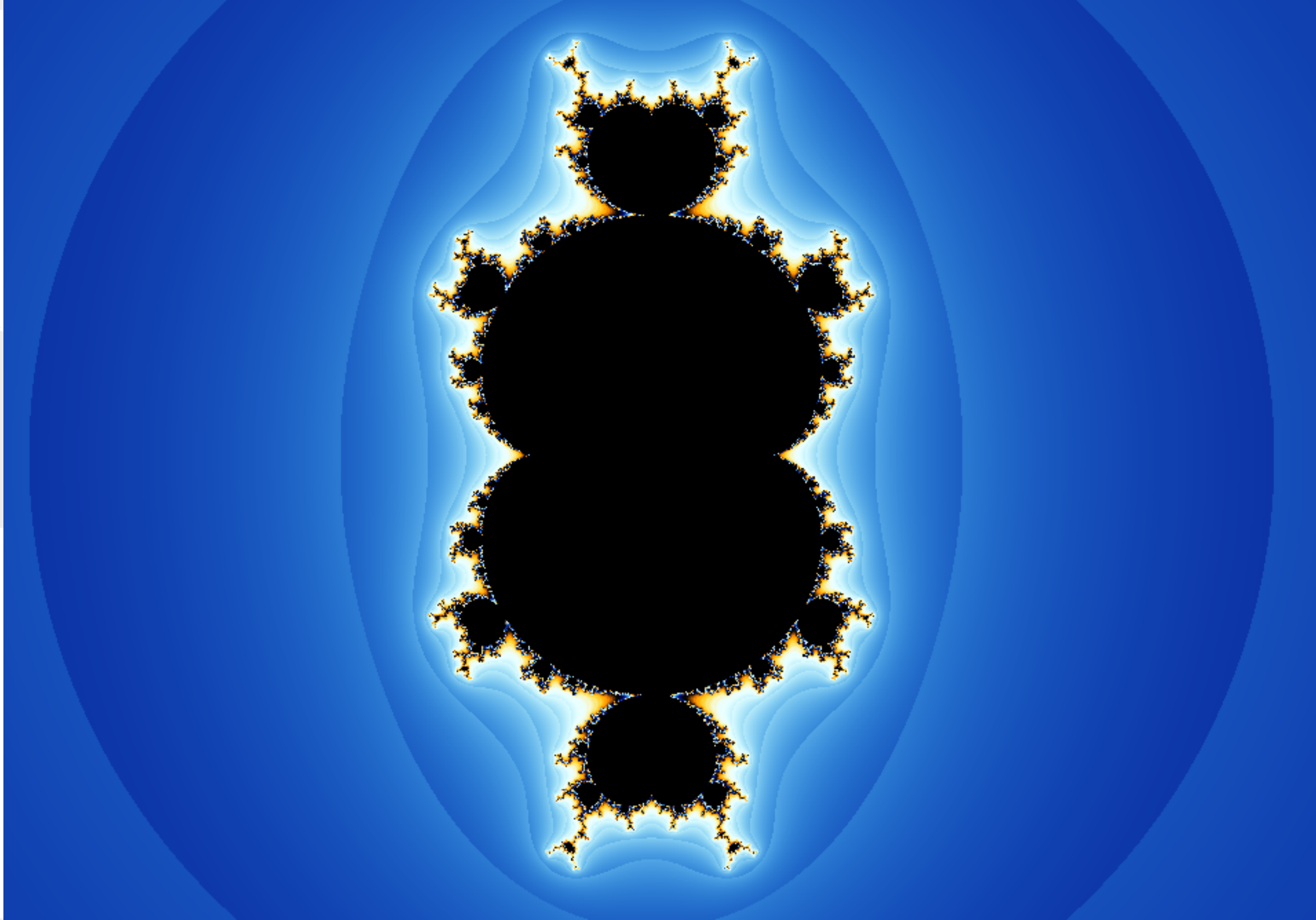


z^2 Mandelbrot Render

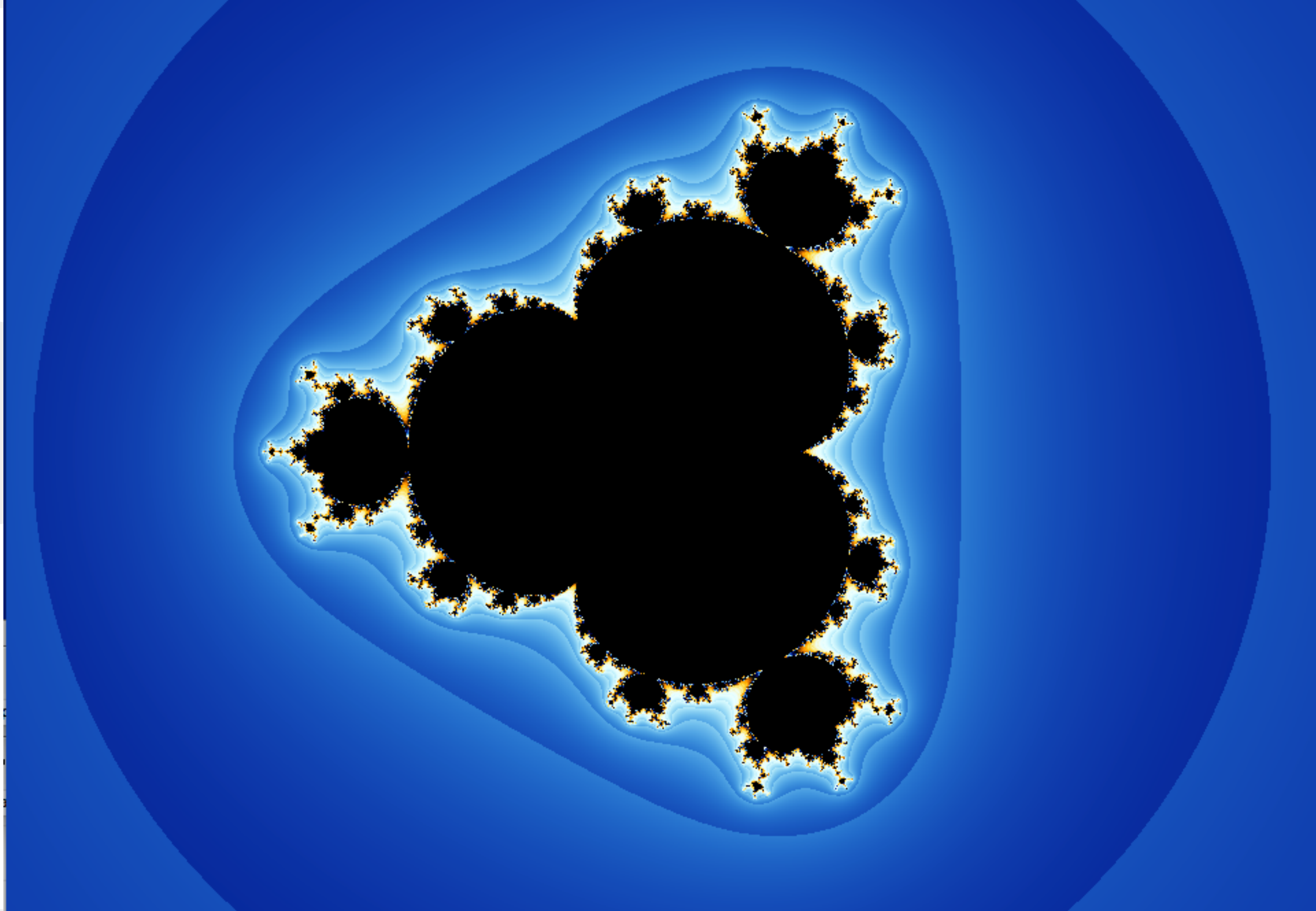
Generic Mandelbrot set.
init:
  z = @start
loop:
  z = z^@power + #pixel
bailout:
  |z| <= @bailout



z³ Mandelbrot Render

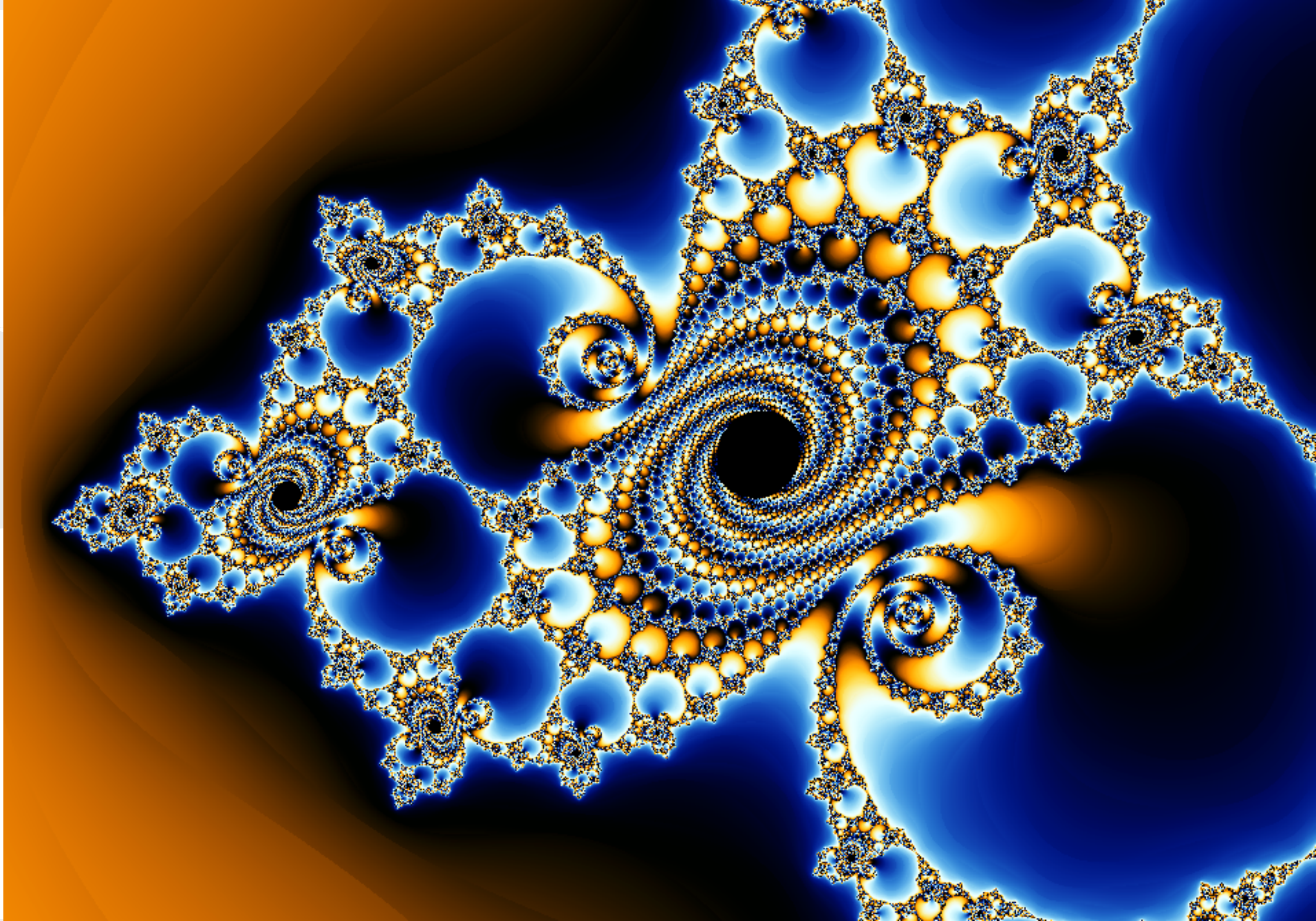


z^4 Mandelbrot Render



1.36×10^{16}
Magnification

Extra:
If the original z^2 Mandelbrot render were 1m wide, this would be smaller than the diameter of a proton.



Sources

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http://fractalfoundation.org/fractivities/FractalPacks-EducatorsGuide.pdf

http://en.wikipedia.org/wiki/Burning_Ship_fractal

https://www.math.ucdavis.edu/~thomases/W11_16C1_lec_3_11_11.pdf

http://en.wikipedia.org/wiki/Mandelbrot_set