History of Complex Analysis

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- Quadratic and cubic equations lead to the issue of square roots of negative numbers
  - Gerolamo Cardano (1501-1576)
    - Italian Renaissance mathematician
    - Introduced complex numbers into algebra a + $\sqrt{}$-b
    - trying to divide 10 into two equal parts whose product is forty
    - (5 + $\sqrt{}$-15)*(5 - $\sqrt{}$-15) = 25 - -15 = 40
    - Called these numbers "fictitious" because they didn't have any meaning
  - Rafael Bombelli (1526-1572)
    - Italian mathematician
    - Introduces notation $\sqrt{}$-1 called *pi´u di meno*
    - solving the equation x^3 = 15x + 4 observing that a solution is x = 4
    - x = $\sqrt[3]{}$(2+$\sqrt{}$-121) + $\sqrt[3]{}$(2-$\sqrt{}$-121)
    - broken down to x = (2 + $\sqrt{}$-1) + (2 - $\sqrt{}$-1)
    - x = a + bi + a - bi = 2a = 2(2) = 4
- Views on numbers
  - Rene Descartes (1596-1650)
    - French philosopher
    - First to use the word imaginary in 1637
    - "For any equation one can imagine as many roots [as its degree would suggest], but in many cases no quantity exists which corresponds to what one imagines."
  - John Wallis (1616-1703)
    - English mathematician
    - believed negative numbers should exist
    - negative numbers is the distance to the left from zero
    - invented the number line
  - Leopold Kronecker (1823-1891)
    - German mathematician
    - "God made the integer, all else is the work of man"
- Further contributions
  - Abraham de Moivre (1667-1754)
    - Frenchman who fled to England where he met Isaac Newton.
    - Developed a formula used by Newton into Moivre's theorem:
    - $(\cos(\theta) + i\sin(\theta))^n = \cos(n\theta) + i\sin(n\theta)$
    - Newton used this formula to solve cubic equations.

- o Leonhard Euler (1707-1783)
  - Born in Switzerland, spent most of his career in St. Petersburg and Berlin
  - Wrote 886 papers and books, approximately 90 volumes.
  - First introduced the notation i
  - Beginnings of the complex plane
  - Euler's formula: $e^{i\theta} = \cos(\theta) + i\sin(\theta)$
- o Jean-Robert Argand (1768-1822)
  - Parisian bookkeeper with unknown mathematical training.
  - Published a pamphlet called "Essay on the Geometrical Interpretation of Imaginary Quantities"
  - Adrien-Marie Legendre got his hands on a copy. He told Francois Francais about it in a letter. When Francois Francais died his son Jaques inherited his papers who used the ideas originally in the pamphlet to publish an article in 1813 which gave the basics for complex numbers.
- o Carl Gauss (1777-1855)
  - German mathematician
  - First used the term complex
  - First discovered what we know as the Cauchy-Riemann equations but was left unpublished
- o Augustin Louis Cauchy (1789-1857)
  - Fled Paris as a child due to turmoil during the French revolution, later worked for Napoleon
  - Initiated complex function theory in 1814, published in 1825
  - First to use the concept of analytic functions
  - Cauchy Integral Theorem- Integrating any complex function around a curve gives zero provided that the function if differentiable everywhere inside the curve.
- o Bernhard Riemann (1826-1866)
  - German mathemetician
  - Cauchy-Riemann equations
  - Riemann Sphere is the extended complex plane
  - Complex plane with a point at infinity allows for division by zero
  - Extended complex number denoted **C** U {∞}

REFERENCES

- http://www.clarku.edu/~djoyce/complex/cubic.html

- http://moodle.technion.ac.il/pluginfile.php/389753/mod_resource/content/1/Merino%202006%20A%20short%20history%20of%20complex%20numbers.pdf

- http://www.math.snu.ac.kr/~ckhan/mathhistory/complexhistory/complexhistory091008.pdf

- http://nrich.maths.org/5961

- http://www-history.mcs.st-and.ac.uk/Biographies/Cauchy.html