Riemann Surfaces

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Bernard Riemann

- German
- Contributions to: analysis, number theory, and geometry
- Developed uniformization theorem

http://en.wikipedia.org/wiki/Bernhard_Riemann

Definition & Uniformization Theorem

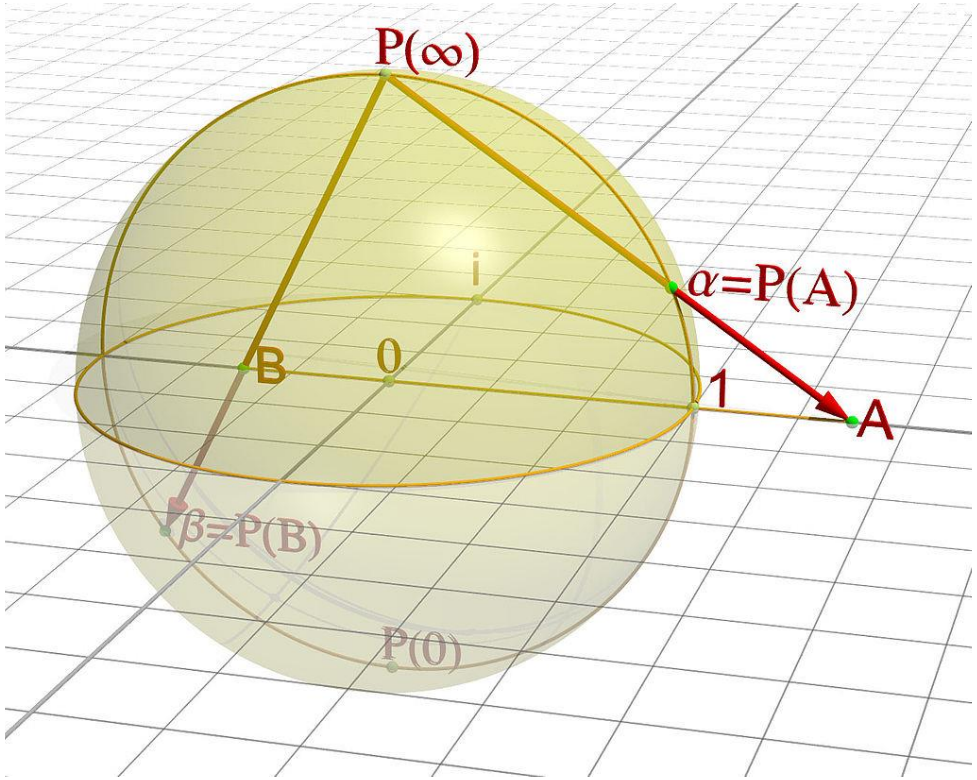
Definition: A Riemann surface is a one dimensional Complex manifold

"a surface-like configuration that covers the complex plane"—Wolfram Math World

- Elliptic (Riemann Sphere): $\mathbb{C} \cup \{\infty\}$ also $P^1\ \mathbb{C}$
- Parabolic: Complex Plane $\mathbb{C}$
- Hyperbolic: the open unit disk $\{z \in C\colon |z| < 1\}$

Elliptic

- The Riemann sphere
- Elliptic geometry
- Compact
- Curvature of +1



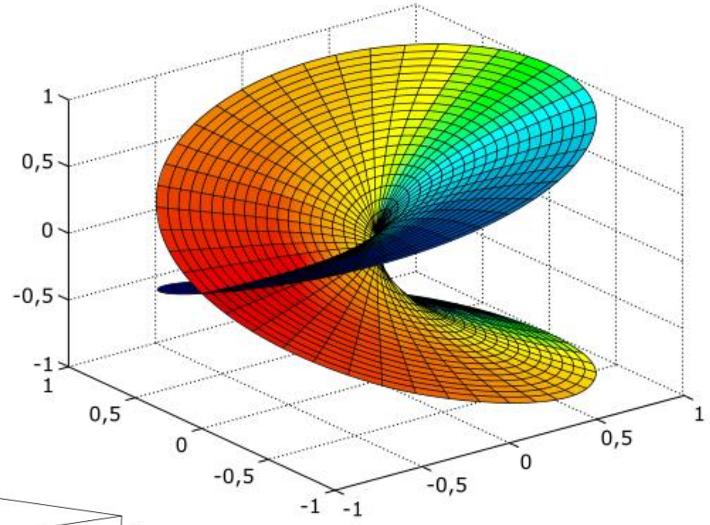
http://en.wikipedia.org/wiki/Riemann_sphere

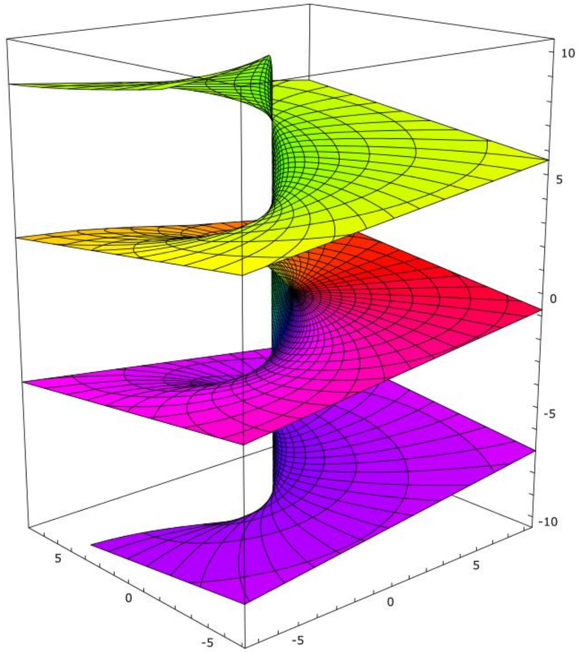
Parabolic

- Isomorphic to $\mathbb{C}$ plane
  - Identity map f(z)=z
  - Conjugate map $f(z)=z^*$

- Not compact

- Curvature of 0
  (Euclidean Geometry)

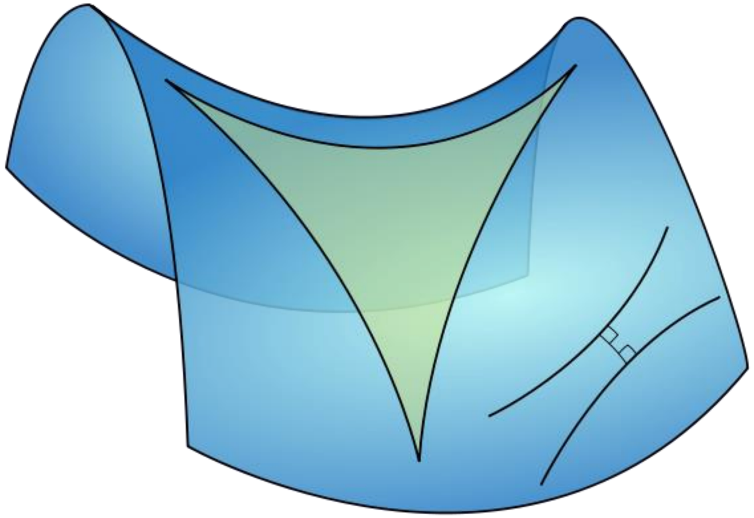




http://en.wikipedia.org/wiki/Riemann_surface#Definitions

Hyperbolic Riemann Surfaces

- Hyperbolic: the Riemann surfaces with curvature -1.
- According to the Uniformization theorem, all hyperbolic surfaces are quotients of the unit disk.
- Unlike elliptic and parabolic, no classification of the hyperbolic surfaces is possible.

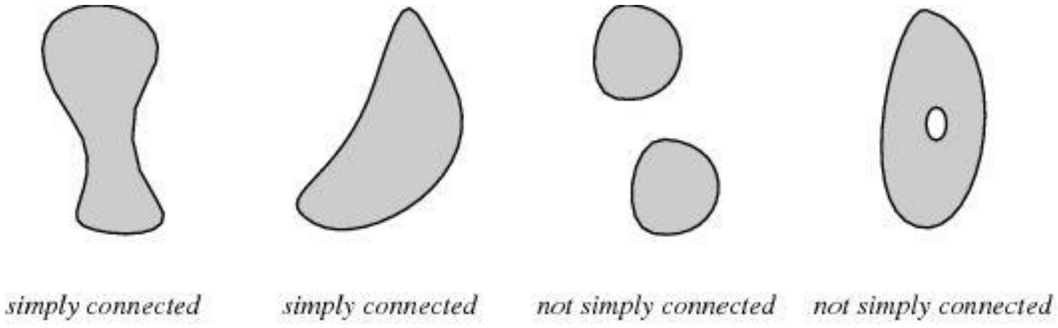


http://en.wikipedia.org/wiki/Hyperbolic_geometry

Riemann Mapping Theorem

- Any open, simply connected region is isomorphic to the open unit disk.

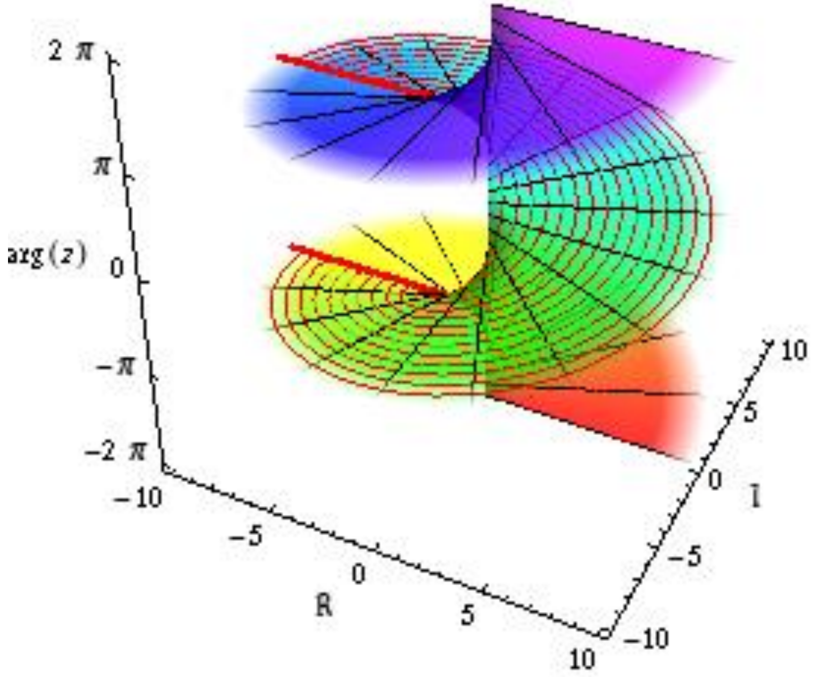
- Isomorphism f(z)=w
  - Holomorphic
  - Unique



http://mathworld.wolfram.com/SimplyConnected.html

Relation to Branching Structures

- The branching theorem assumes a Riemann surface
- Branch points can be thought of as punctures in the initial Riemann sphere.
  - 0 gives sphere
  - 1 or 2 gives complex plane or cylinder
  - 3 or more gives hyperbolic structures



http://en.wikipedia.org/wiki/Argument_(complex_analysis)

Resources

- Brown, J., & Churchill, R. (2013). Complex Variables and Applications (9th edition). New York, NY: McGraw-Hill Science/ Engineering/ Math.
- http://en.wikipedia.org/wiki/Hyperbolic_space
- http://en.wikipedia.org/wiki/Riemann_surface#Classification_of_Riemann_surfaces
- http://mathworld.wolfram.com/RiemannMappingTheorem.html
- http://en.wikipedia.org/wiki/Hyperbolic_geometry
- http://mathworld.wolfram.com/SimplyConnected.html
- #http://en.wikipedia.org/wiki/Riemann_surface#Definitions
- http://en.wikipedia.org/wiki/Argument_(complex_analysis)