Math 360 Spring 2015 Final Exam

Tuesday, May 5, 2015

Honor Pledge: I understand that it is a violation of the JMU honor code to give or receive unauthorized aid on this exam. Furthermore, I understand that I am obligated to report any violation of the honor code by other students that I may become aware of, and that my failure to do so is itself a violation. No phones, or other electronic devices, other than a calculator, may be accessed during this test. Doing so will be considered a violation of the honor code.

Name: ______________________________

Signature: ______________________________

1 Solve the following to the best of your knowledge

1. Solve the equation $z^3 + 1 = 0$, and plot the roots in the complex plane.

2. Show that the equation $z^6 + 3z^4 + 1 = 0$ has two roots that lie in the upper half of the unit circle. (Hint: Both Rouche's theorem and the reflection principle are involved here.)

3. (a) Find the radius of convergence of the series $f(z) = \sum_{n=1}^{\infty} \frac{z^n}{n}$.

   (b) This series is the integral of a well known series. Use that and analytic continuation to find the value of the function $f(z) = \sum_{n=1}^{\infty} \frac{z^n}{n}$ at $z = 2i$.

4. Find the electrostatic potential V in the space enclosed by the half circle $x^2 + y^2 = 1, y \geq 0$ and the line $y = 0$ when $V = 0$ on the circular boundary, and $V = 1$ on the line segment $[-1, 1]$. Justify all your steps. (Hint: Conformally map the upper half unit disk to an easier domain.)

5. (a) State Cauchy's theorem and explain why it is so important in our complex analysis class.

   (b) Suppose that $f(z)$ is entire and has n simple zeros at $z_1, z_2, \ldots, z_n$. Suppose also that $|f(z)| \leq k|z|^m + L$ for some m, k and L. Show that $f(z)$ is a polynomial of degree and form that you should be able to determine. (Hint: Use Cauchy's estimate and let $R \to \infty$ since the function is entire.)

6. (a) Expand the function $f(z) = \frac{1}{z(z-1)(z-2)}$ in powers of z valid in the annulus $1 < |z| < 2$. Can you determine the residue of $f(z)$ at $z = 0$ from this expansion? Or will you need to find another expansion in a different domain?

   (b) Use a different method than the Laurent series to find $Res(f(z); z = 0)$.

7. Find the real and imaginary parts of the number i^i.

8. Use the contour C which is the union of the line segment $[-R, R]$ on the x-axis, and the semicircular arc $C_R = \{z : |z| = R\}$ in the upper half plane to show that the integral $\int_0^\infty \frac{\cos(2x)}{(x^2+4)^2}\,dx = \frac{5\pi}{32e^4}$. Justify all your steps (for your estimate, Jordan's lemma should do the trick).

9. Define a complex analytic function, and state all its properties that you can recall.

10. Explain, in your own words, what you learned the most in this Complex Analysis course. Don't forget to have a good summer afterwards!