Math 360 Spring 2015 Midterm Exam

March 2015

Honor Pledge: I understand that it is a violation of the JMU honor code to give or receive unauthorized aid on this exam. Furthermore, I understand that I am obligated to report any violation of the honor code by other students that I may become aware of, and that my failure to do so is itself a violation. No phones, or other electronic devices, other than a calculator, may be accessed during this test. Doing so will be considered a violation of the honor code.

Name: _____

Signature: _____

1 Solve the following to the best of your knowledge

1. Solve the equation $z^4 = -16$, and plot the roots in the complex plane.

2. (a) What is the image of the upper half plane under the transformation $f(z) = z^3$? Is this transformation one-to-one? Discuss and use graphs and polar coordinates if necessary.

   (b) Explain why the function e^z is periodic, and discuss the image of the complex plane under the exponential transformation.

3. (a) Find the harmonic conjugate $v(x, y)$ of the function $u(x, y) = y^2 - x^2$ in the complex plane. Deduce the analytic function $f(z) = u + iv$ in terms of z. Plot the level sets $u(x, y) = c_i$ and $v(x, y) = c_j$ and comment on their orthogonality.

(b) Find $f'(z)$.

4. Explain the branch cut structure of the function $f(z) = \sqrt{z^2 - 1}$ (Hint: This is the product of two square root functions). Is the point at infinity a branch point? Why or why not?

5. Find all functions $f(z)$ satisfying all the following properties:

   (a) $f(z)$ is analytic on $\{\Im(z) > 0\}$,

   (b) $f(z)$ is continuous on $\{\Im(z) \geq 0\}$,

   (c) $f(z)$ is real on the real axis,

   (d) $|f(z)| > |\cos(z)|$ on $\{\Im(z) > 0\}$.

6. Find the real and imaginary parts of the number $(1+i)^\pi$.

7. Compute the integral $\int_\gamma \bar{z}dz$, where:

(a) γ is the contour from $z = 0$ to $z = 1$ to $z = 1 + i$.

(b) γ is the unit circle (with center 0 and radius $R = 1$). Hint: Use polar coordinates to parametrize γ.

Is the above integral path independent? Why or why not?