Math 360 Complex Variables (Spring 2015) Week 1 Worksheet

Week Highlights

1. The difference between real analysis and complex analysis.
2. Real smooth function (C^∞) vs real analytic function.
3. The strength of differentiability (on an open neighborhood of z_0) for a complex function: (implies both smoothness and analyticity).
4. The easiness and beauty of complex analytic functions (do you already know some properties? Uniquely determined analytic functions, *etc.*).
5. The complex plane (Argand plane) $(\mathbb{C}, +, \times)$, complex numbers ($z = x+iy$), their conjugates ($\bar{z} = x - iy$), moduli ($|z| = \sqrt{x^2+y^2}$), and polar form ($z = |z|e^{i\arg(z)} = re^{i\theta}$). (There is no order on $\mathbb{C}$!)
6. Multivalued $\arg z$ and single valued Principal Argument function $Arg : \mathbb{C} \to (-\pi, \pi]$ (discontinuous along the negative real axis, because of a jump of value 2π overthere.)
7. Similarities and *differences* between $\mathbb{C}$ and $\mathbb{R}^2$.
8. Formulas:
   (a) Euler's Formula: $e^{i\theta} = \cos\theta + i\sin\theta$.
   (b) De Moivre's formula: $e^{in\theta} = (\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$.
9. Inequalities:
   (a) $\Re(z) \le |\Re(z)| \le |z|$.
   (b) $\Im(z) \le |\Im(z)| \le |z|$.
   (c) $|z_1 \pm z_2| \le |z_1| \pm |z_2|$.
   (d) $|z_1 \pm z_2| \ge ||z_1| - |z_2||$.

Reading assignment: Read Chapter 1 from the book.
Problem Set Hand the following problems.

1. List all the differences that you know between complex analysis and real analysis.
2. List all the similarities and differences that you know between $\mathbb{C}$ and $\mathbb{R}^2$.
3. What is the difference between a smooth (C^∞) function and an analytic function:
   (a) in $\mathbb{R}$?
   (b) in $\mathbb{C}$?

(Note that the answer of number 3 is included in number 1, but you can elaborate more here.)