Infinities in Complex Plane

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Motivation

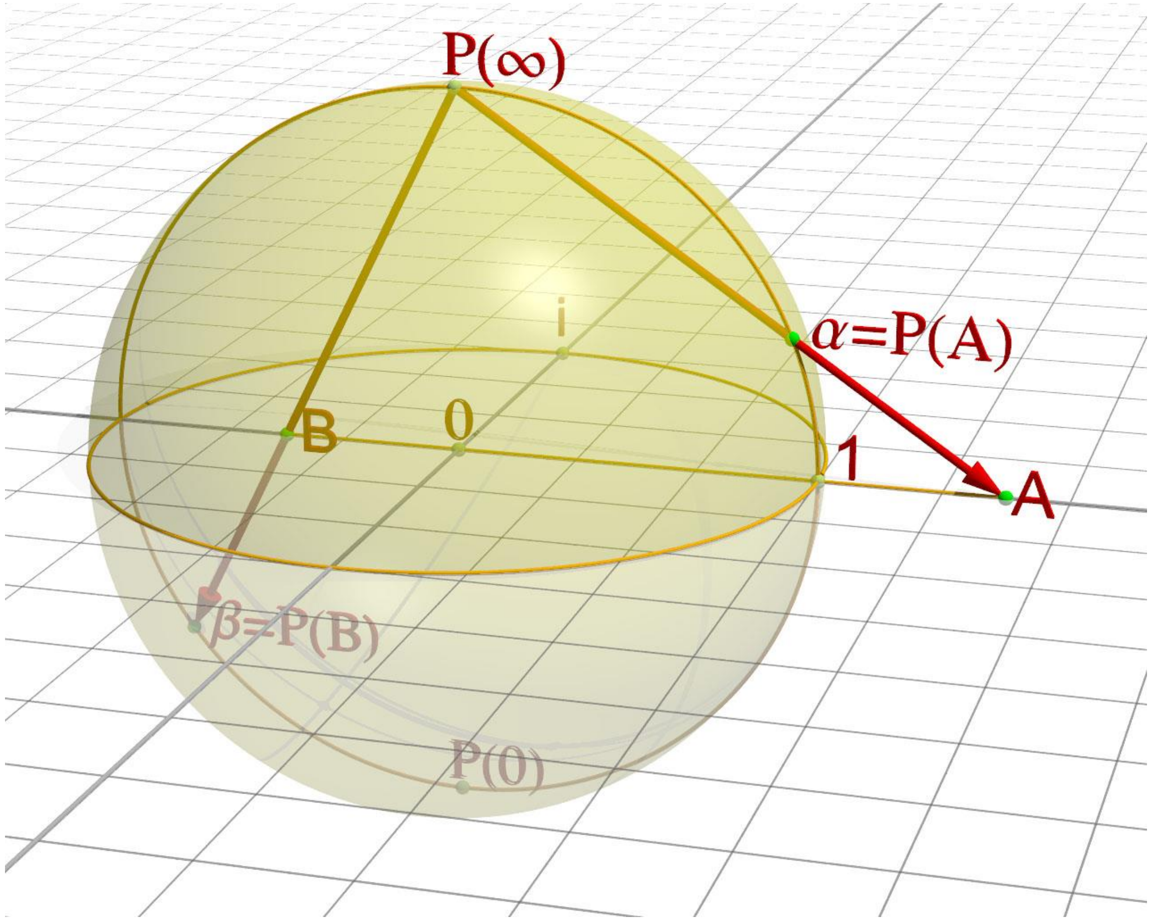
- The concept of infinity is clear in R since R has *order*. It is clear when a number is less than or greater than another number.
- In R we can imagine a function that just keeps increasing to arbitrarily large number like f(x)=x^2.
- C does not have order. Is 1+2i greater or less than 2-i?
- What does it mean to be infinite in C.

Norm to the Rescue

- We can define norm for a complex function f, |f|, to impose something on C that we can order. As |f| grows arbitrarily large you can imagine a circle who's radius grows as |f| increases.
- Does this mean there are 1 or many infinities in the complex plane?
- A sphere can help us.

Riemann Sphere

- Use a projection called stereographic projection to map points on the sphere onto C
- Where does the North Pole go?
- Only 1 infinity in C (with the Riemann sphere mapping)



Consequences?

- Define the *extended* complex number. Just C with infinity smashed in.
- $C\infty$ is no longer a field since ∞ has no multiplicative inverse.
- We can define other cool structure though, unlike R the following are defined in C_∞:
  - $z/0=\infty$
  - $z/\infty=0$
  - $\infty/0=\infty$
  - $0/\infty=0$

$0/0$ and ∞/∞ are still undefined ☹

Resources

- Brown, J., & Churchill, R. (2013). *Complex Variables and Applications* (9 edition). New York, NY: McGraw-Hill Science/Engineering/Math.
- http://en.wikipedia.org/wiki/Riemann_sphere
- http://plus.maths.org/content/maths-minute-riemann-sphere