

Math 441 Analysis and Dynamics of Differential Equations

1 Assignment on ODE theory, existence and uniqueness

Read your lecture notes on existence and uniqueness, and the same topics from any ODE theory book. Submit the following problems.

1. Prove the uniqueness part of Picard's Theorem.
2. Use Picard's iteration to solve the IVP $y' = 2t - y$, $y(0) = 1$. Find the interval of convergence of

the obtained series, and an analytical formula of the solution it converges to. Plot the iterates and the solution on the same graph (using Matlab or some other graphing utility), illustrating the convergence.

3. Dropping the Lipschitz condition on f in $x' = f(t, x)$ may cost uniqueness. Illustrate using an example. Prove existence of solutions without the Lipschitz condition.