Nonlinear Dynamical System in Humanoid Robots

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Background

- Paper was written by Jan Ijspeert, Jun Nakanishi and Stefan Schaal
- Created a Dynamical System for autonomous robotic movements
  - Models arm movements
- Dynamical System works as a Control Policy which sends messages to the controller of the humanoid robot
- Movement is represented in Kinematic movements

Dynamical System

$$\dot{z} = \alpha_z(\beta_z(g - y) - z)$$

$$\dot{y} = z + \frac{\sum_{i=1}^{N} \psi_i \omega_i}{\sum_{i=1}^{N} \psi_i} v$$

- This system is our Control Policy
- g is our goal state
- Y is our desired state
- Constants α_z and $\beta_z > 0$
- Describes a trajectory y towards a goal g

Dynamical System Cont.

$$\dot{v} = \alpha_v(\beta_v(g - x) - v)$$

$$\dot{x} = v$$

- This system describes the internal state of the robot
- Essential to determining the velocity
- Constants α_z and $\beta_z > 0$
- X is used to localize the Gaussian Kernel in the first system
- V is a scaling term used to ensure the system will have a unique attractor

Gaussian Kernel

$$\frac{\sum_{i=1}^{N} \psi_i \omega_i}{\sum_{i=1}^{N} \psi_i} \; where\; \psi_i = e^{-\frac{1}{2\sigma_i^2}(\tilde{x}-c_i)^2}$$

- Used for machine learning and prediction
- ω_i found using locally weighted regression

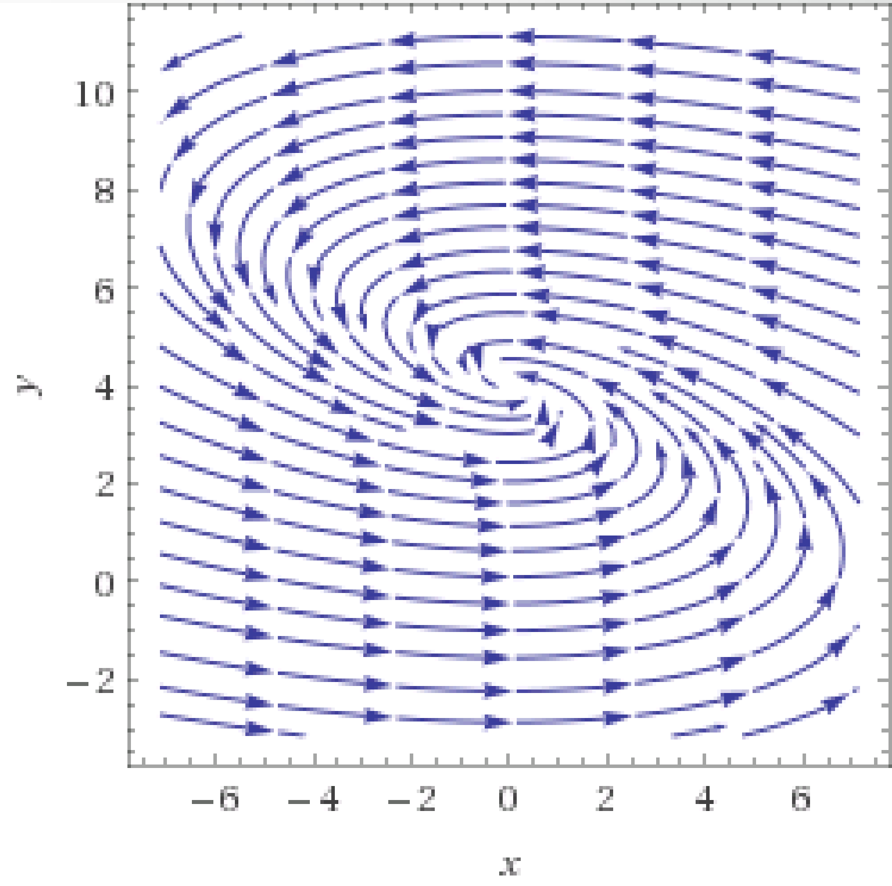
Stability

- We have one fixed point for (z,y) at (0,g)
- Next we find the Jacobian Matrix: $\begin{bmatrix} -\alpha_z & -\alpha_z\beta_z \\ 1 & 0 \end{bmatrix}$
- Then we evaluate the Jacobian Matrix at (0,g) and find the eigenvalues
- Our eigenvalues are: $\lambda_1 = \frac{1}{2}\left(-\sqrt{\alpha_z} * \sqrt{\alpha_z - 4\beta_z} - \alpha_z\right)$

$$\lambda_2 = \frac{1}{2}\left(\sqrt{\alpha_z} * \sqrt{\alpha_z - 4\beta_z} - \alpha_z\right)$$

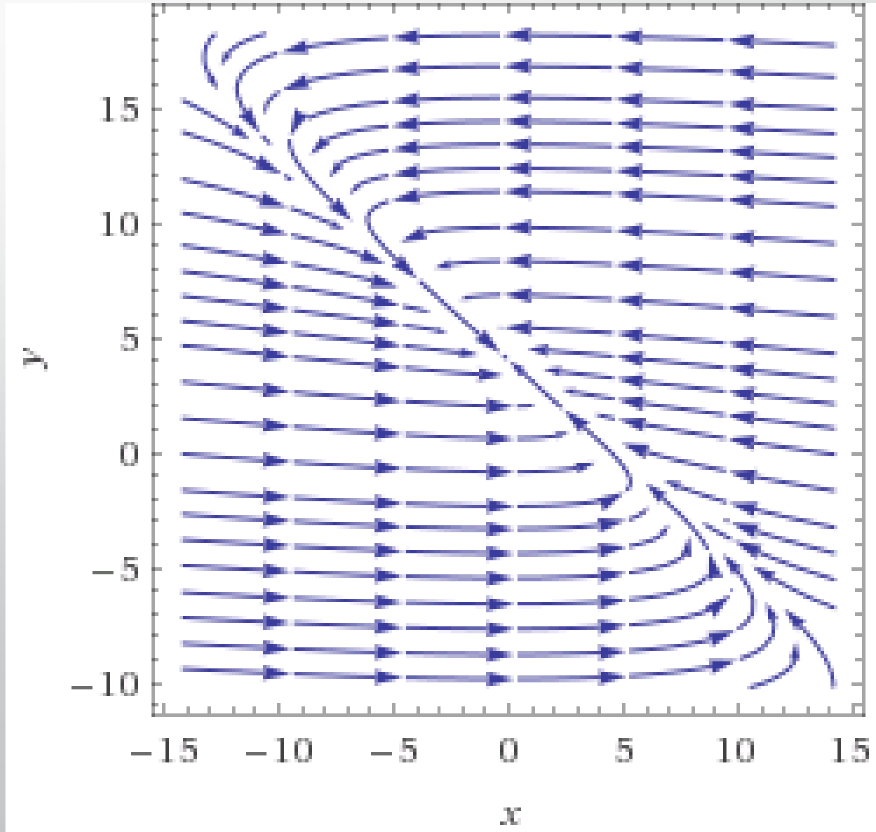
Stability – Imaginary Eigenvalues

- When $\alpha_z < 4\beta_z$ we get imaginary eigenvalues
- Phase Portrait will spiral in
- Thus we have stability at (0,g)



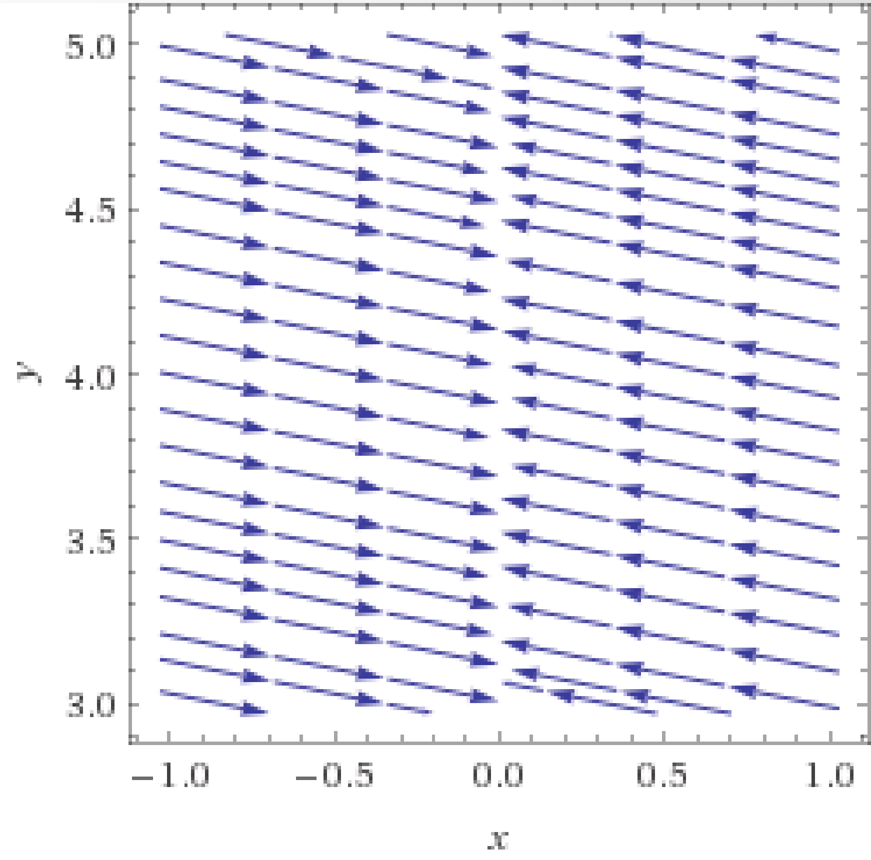
Stability – Real Eigenvalues

- When $\alpha_z > 4\beta_z$ we get real eigenvalues
- When $\alpha_z > \sqrt{\alpha_z}\sqrt{\alpha_z - \beta_z}$ we get one eigenvalue < 0 while the other is > 0
- Thus our phase portrait is a saddle point that is stable



Stability – Real Eigenvalues

- When $\alpha_z = \sqrt{\alpha_z}\sqrt{\alpha_z - \beta_z}$ then one of our eigenvalues equals 0
- This gives us an infinite line of fixed points



Conclusion

- Fixed point (0,g) is both globally attracting and Liapunov stable so we call it asymptotically stable
- Dynamical System always achieves its goal state
- Created this system to help stroke patients