Calculating the Lorenz System's Lyapunov Exponents

The Caculation of Chaos

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Outline

The Lorenz System

Lorenz started working with idealized hydrodynamical systems in order to study convection and other weather related events in 1963.
His equations are derived from Saltzman's equations studying convection.

$$x' = \sigma y - \sigma x, \tag{1}$$

$$y' = rx - y - xz, \tag{2}$$

and

$$z' = xy - bz. \tag{3}$$

Basic Aspects of Lorenz

r is our parameter.

$$r = 0$$

$$0 < r < 1$$

$$1 < r < r_H$$

$$r > r_H$$

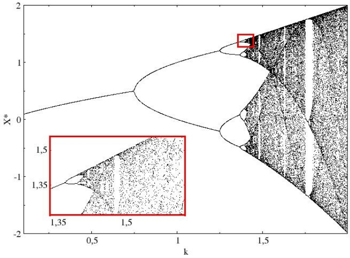

Figure: Biffurcation diagram of the Lorenz equations. Please ignore the k value at the bottom.

Lyapunov Exponents

The average rate of divergence or convergence of nearby orbits in space (Wolf 1985). The signs of the spectrum of Lyapunov exponents helps define a dynamical system. We can find the largest Lyapunov exponent

$$\lambda_i = \lim_{t \to \infty} \lim_{\delta Z_0 \to 0} \frac{1}{t} ln \frac{\delta Z(t)}{\delta Z_0}$$

Calculating Lyapunov exponents takes a lot of time and computing power.

Wolf's 1985 Paper

The aim was to devise a way to determine if a set of time series data was chaotic or not.

Current methods were not accurate or effective.

Studied known chaotic dynamical systems to figure out ability of program to determine Lyapunov exponents using Grahm-Schmidt Reorthonormalization.

Grahm-Schmidt Reorthonormalization

1. Start with an initial condition
$Y = [x, y, z, \delta_{xx}, \delta_{xy}, \delta_{xz}, \delta_{yx}, \delta_{yy}, \delta_{yz}, \delta_{zx}, \delta_{zy}, \delta_{zz}]$
2. Allow that point to grow in time
3. Use Grahm-Schmidt Reorthonormalization process
4. Save the information of the growth in the δ_x,δ_y,δ_z directions.
5. Use the growth information to calculate the Lyapunov exponents

References

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