# DYNAMICAL BILLIARDS

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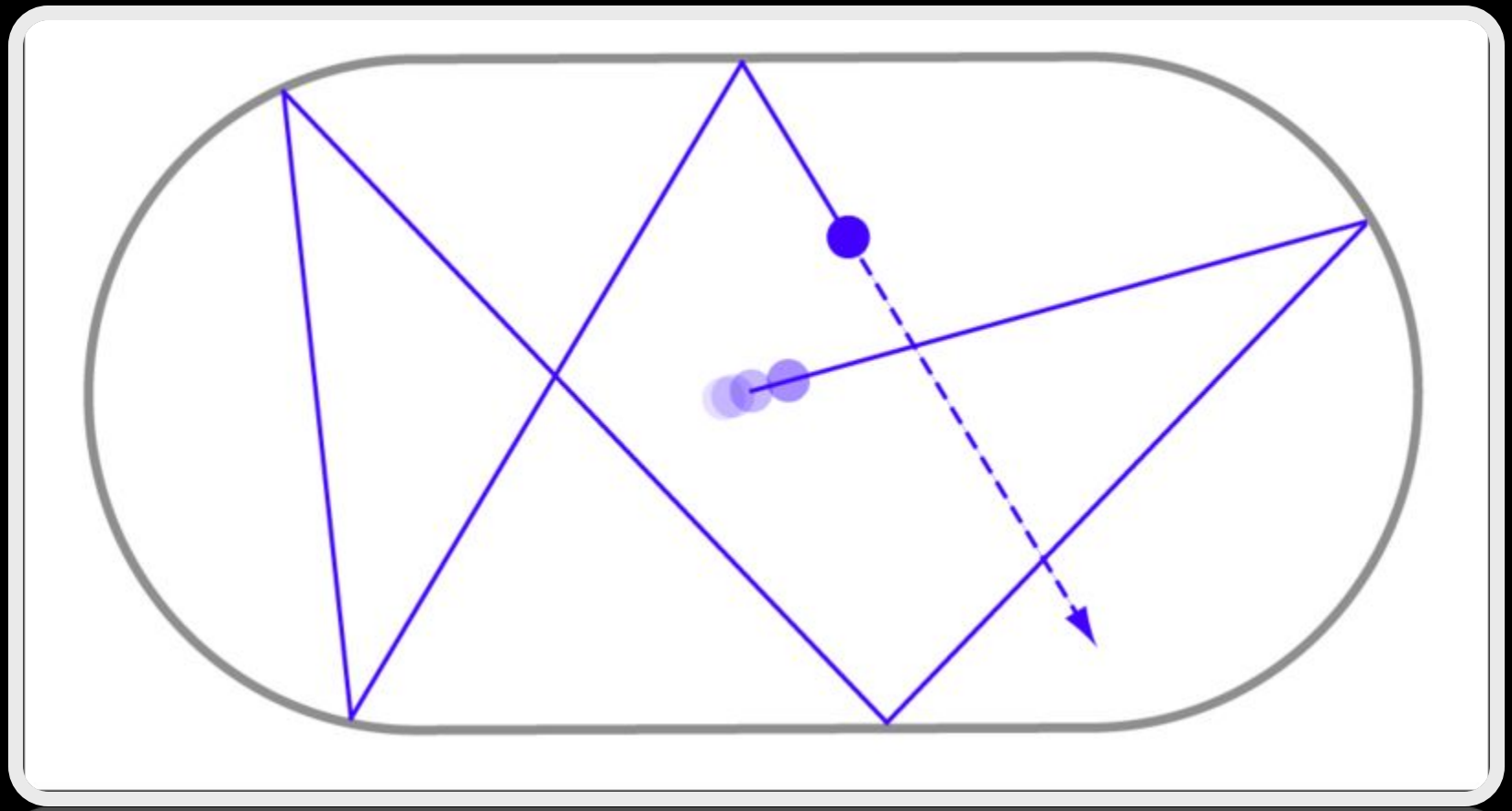
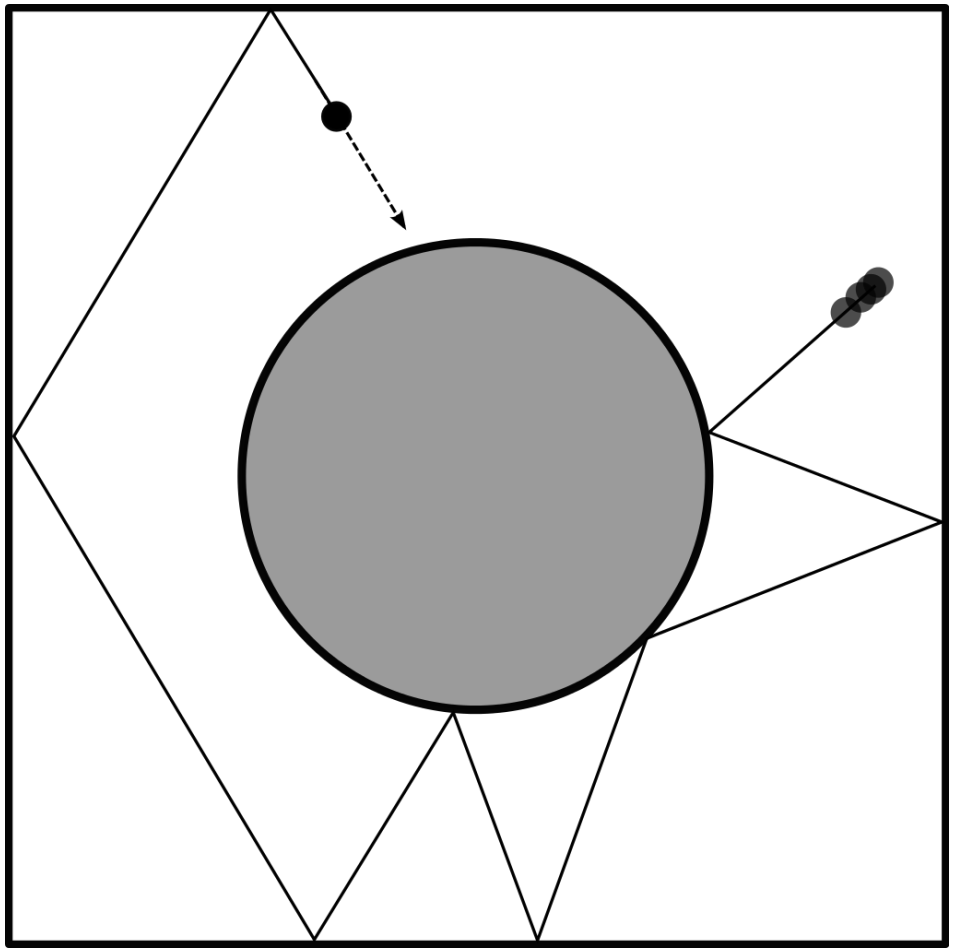
Math 441

# BACKGROUND INFORMATION

- A system where a particle alternates between motion in straight lines and specular reflections from boundaries without loss of speed.
- Are Hamiltonian idealizations of the actual game of billiards.
- The boundary can have different shapes other than the standard rectangle.





# BACKGROUND CONTINUED

- Captures the complexity of Hamiltonian systems (from integrability to chaotic motion)
  - Does it without the difficulties of integrating the equations of motion to determine its Poincare map.
  - The equation for a particle with mass m with no friction force:

$$H(p,q) = \frac{p^2}{2m} + V(q)$$

  - Where V(q) is the potential inside or outside the region.

# QUANTUM BILLIARDS

- Readily studied with the formula:

$$-\frac{\hbar^2}{2m}\nabla^2\psi_n(q) = E_n\psi_n(q)$$

- With Dirichlet boundary conditions

$$\psi_n(q) = 0 \quad \text{for} \quad q \notin \Omega$$

# QUANTUM BILLIARDS CONTINUED

- Taken a look at the free-field model, we get:

$$\left(\nabla^2 + k^2\right)\psi = 0$$

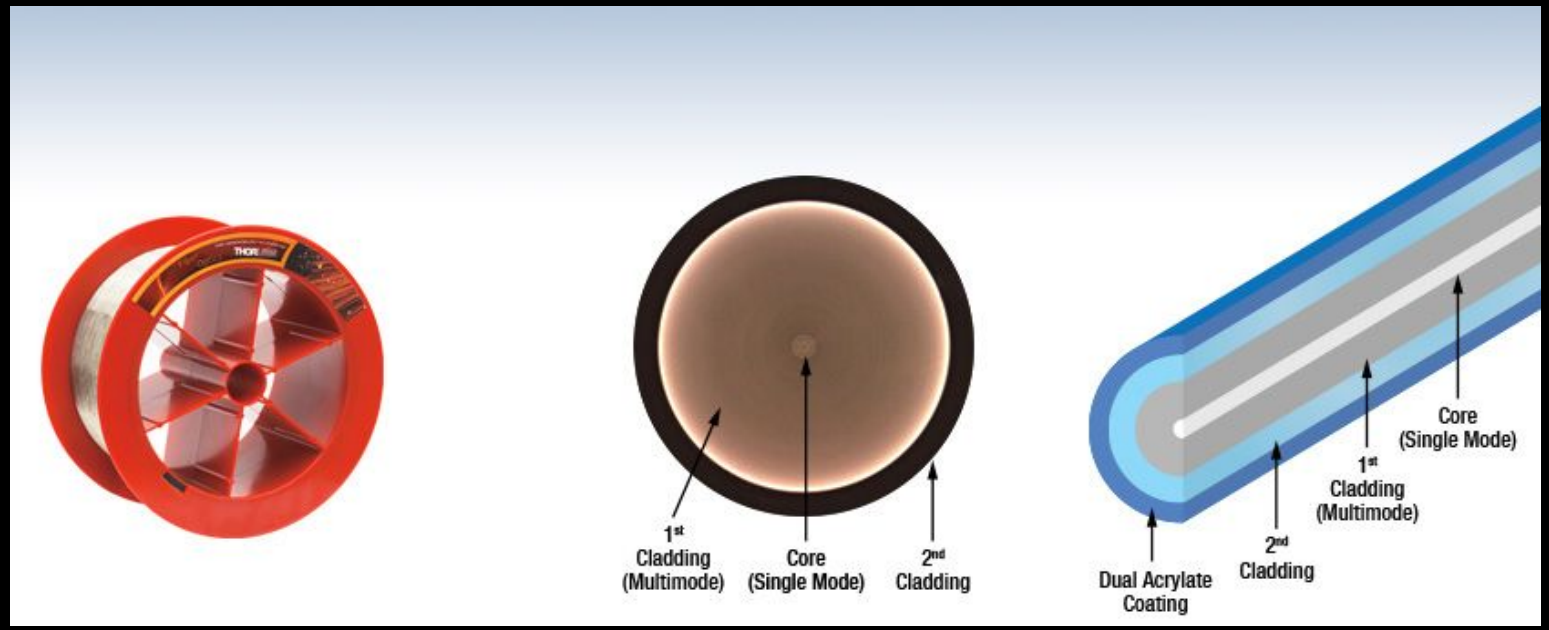
- Where

$$k^2 = \frac{1}{\hbar^2} 2mE_n$$

- (Implies that two and three dimensional quantum billiards can be modelled by classical resonance modes of a radar cavity of the given shape.

# APPLICATIONS

- Most common application is with double-clad fibers



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