Introduction to Chaos of Financial Markets

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Overview

Motivation

- The stock market may be governed by chaotic dynamics
- Chaotic dynamics can randomly simulate unexpected bursts of volatility and large movements in the stock market
- Chaos is helpful in explaining fluctuations in the economy and financial markets
- Existence of chaotic behavior in financial markets is widely debated

Brief Review of Fundamental Concepts

Dynamical System

A way of describing the passage in time of all points of a given space S.

Dimension of a Dynamical System

The number of state variables that can specify a point in the system.

Chaos

A dynamical system is described as chaotic if it is at least three dimensional and extremely sensitive to initial conditions.

Low Complexity Chaos

- Other types of chaos are uninteresting for individuals who want to understand how to predict the pattern of financial markets
- Detecting low complexity chaos for relatively small numbered dimensions entails the use of chaotic maps such as the Tent Map and Lorenz Map.
- Autocorrelation can affect the testing for low complexity chaos which is why it must be removed from one's dataset

Mackey-Glass Equation

Why is this Important?

The Mackey-Glass Equation is a delayed differential equation commonly used to detect high dimensional chaotic systems. Note that for n-dimensional chaotic systems, it becomes more difficult to test for chaos as n becomes larger.

$$\frac{dy}{dt} = \frac{ax(t-c)}{1+x(t-c)^10} - bx$$

$$a = 0.2, b = 0.1, c = 100$$

Real-World Example: Chaos in Exchange Rates

$$S_t = \alpha(\frac{e^e}{e_t} - 1), \alpha \geq 0$$
$$T_t = \beta(e_t - e^e) + \gamma(e_{t-1} - e^e)\beta, \gamma > 0$$
$$\nabla S_t = T_t$$

$$e_t = \text{domestic price of foreign currency}$$
$$e^e = \text{future estimated exchange rate}$$
$$\alpha = \text{sensitivity parameter}$$
$$S_t = \text{demand for foreign currency}$$
$$T_t = \text{trade balance}$$

Conclusion

Today, we discussed:

- The relevance of chaos for financial markets.
- Why only one type of chaos is significant when studying market movements.
- The role of the Mackey-Glass Equation in detecting chaos.
- An example of a exchange rate model and it's chaotic behavior.

Thank You!