Math 441 (Spring 2017) Assignment One

1. (Dynamical system) Give the mathematical definition of a dynamical system (differential equations approach).

2. (Pendulum) In class we worked out the nonlinear pendulum example. Consider the linear pendulum. Construct the phase portrait and provide its physical interpretation. Contrast your results with those corresponding to the nonlinear pendulum.

3. (Logistic equation) Find the analytical solution of the logistic equation with constant harvesting

$$x' = x(1 - x) - h$$

for all values of the parameter $h > 0$. Plot the solution for different values of h using a graphing utility. Is there bifurcation? If yes, draw a bifurcation diagram.

4. (Analytical solution) Consider the first order linear nonautonomous differential equation (hence two dimensional dynamical system) of the form

$$x'(t) = a(t)x(t).$$

Find a formula involving integrals for the solution of this system.

5. (One dimensional phase portrait) Find all the stationary solutions of this differential equation:

$$x' = |1 - x^2|,$$

and determine whether they are sinks, sources or neither. Plot the phase portrait.

6. (Damped harmonic oscillator, example of a two dimensional dynamical system) For the harmonic oscillator system

$$x'' + bx' + kx = 0,$$

find all values of b and k for which this system has real, distinct eigenvalues. Find the general solution of this system in these cases. Find the solution that satisfies the initial condition $(0, 1)$, and describe the motion of the mass in that particular case.

7. (Linear algebra) Prove that the eigenvectors corresponding to distinct real eigenvalues are always linearly independent.