## Math 441 (Spring 2017) Assignment Three

1. (Two dimensional phase portrait) Determine the phase portrait of the system $\vec{x}^{\prime}(t)=A \vec{x}(t)$ where $A=\left(\begin{array}{cc}3 & 5 \\ -2 & -2\end{array}\right)$. Find the general solution of the system and the canonical form of the matrix $A$.
2. (Jordan form, repeated eigenvalues) Prove that $\alpha e^{\lambda t}\binom{1}{0}+\beta e^{\lambda t}\binom{t}{1}$ is the general solution of

$$
\vec{x}^{\prime}(t)=\left(\begin{array}{cc}
\lambda & 1 \\
0 & \lambda
\end{array}\right) \vec{x}(t)
$$

Plot the corresponding phase portrait.
3. (Nonlinear system) Plot the phase portrait of the following nonlinear system

$$
\left\{\begin{array}{l}
x^{\prime}=|y| \\
y^{\prime}=-x
\end{array}\right.
$$

4. (Exponential of a matrix) Find the general solution of the linear system in problem two above using the exponential of a matrix. Make sure the solution you obtained in problem two and this one match.
5. (Conservative systems) Strogatz problem 6.5.1.
6. (Reversible systems) Strogatz problem 6.6.3.
