Math 441 (Spring 2017) Assignment Three

- 1. (Two dimensional phase portrait) Determine the phase portrait of the system $\vec{x}'(t) = A\vec{x}(t)$ where $A = \begin{pmatrix} 3 & 5 \\ -2 & -2 \end{pmatrix}$. Find the general solution of the system and the canonical form of the matrix A.
- 2. (Jordan form, repeated eigenvalues) Prove that $\alpha e^{\lambda t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta e^{\lambda t} \begin{pmatrix} t \\ 1 \end{pmatrix}$ is the general solution of

$$\vec{x}'(t) = \left(\begin{array}{cc} \lambda & 1 \\ 0 & \lambda \end{array} \right) \vec{x}(t).$$

Plot the corresponding phase portrait.

3. (Nonlinear system) Plot the phase portrait of the following nonlinear system

$$\begin{cases} x' = |y| \\ y' = -x \end{cases}$$

4. (Exponential of a matrix) Find the general solution of the linear system in problem two above using the exponential of a matrix. Make sure the solution you obtained in problem two and this one match.

1

- 5. (Conservative systems) Strogatz problem 6.5.1.
- 6. (Reversible systems) Strogatz problem 6.6.3.