# Math 441 Analysis and Dynamics of Differential Equations
# Written Assignment 4

**Read your lecture notes and chapters 5 and 6. Submit the following problems. (Topics: Two Dimensional Flows. Linear and nonlinear systems. Phase Portraits.)**

1. The matrix $A = \begin{pmatrix} 4 & -10 \\ 2 & -4 \end{pmatrix}$ has an eigenvalue $\lambda_1 = 2i$ and a corresponding eigenvector $\underline{v} = \begin{pmatrix} 2+i \\ 1 \end{pmatrix}$.
   (a) Find the real solution of the system $\underline{w}' = A\underline{w}$.
   (b) Each trajectory is an ellipse. Write down the parametric equations of these ellipses: $x(t) =$ ... and $y(t) =$ ....
   (c) Along which directions are the major and minor axis of these ellipses are? (Does the eigenvector tell you any information about this?)
   (d) Use a graphing utility to plot the coordinates $x(t)$ and $y(t)$ as a function of $t$, when the initial conditions are $x(0) = -4$ and $y(0) = -1$.
   (e) Plot the phase portrait.
2. Analyze the type and stability of the critical points of the following predator-prey ecological system, then plot a phase portrait showing all interactions and the separatrices if any.
$$\begin{cases} x'(t) = 7x - x^2 - 2xy \\ y'(t) = y - y^2 + 2xy. \end{cases}$$
3. Consider the damped nonlinear pendulum
$$\theta'' + 0.1\theta' + \sin(\theta) = 0.$$
   Transform into a first order system, find the critical points, linearize to study the types and the stability of the critical points, then plot a phase portrait, showing the separatrices.
4. Use a phase portrait graphing utility to plot the phase portraits of the systems in problems 1, 2, and 3 above, hence confirming your predictions.
5. 5.1.10 (a, d), 5.1.11 (a,d).
6. 6.2.1.
7. 6.3.10.
8. 6.3.11.