Math 441 (Spring 2017) Assignment Five

1. (Picard's iteration) For each of the following IVPs, use Picard's iteration to solve the IVP. Find the interval of convergence of the obtained series, and an analytical formula of the solution it converges to. Plot the iterates and the solution on the same graph (using Matlab or some other graphing utility), illustrating the convergence.

   (a) $x' = 2t - x, x(0) = 1$.

   (b) $x' = \frac{1}{2x}, x(1) = 1$.

2. (Existence and uniqueness)

   (a) Prove the uniqueness part of Picard's Existence and Uniqueness Theorem.

   (b) Dropping the Lipschitz condition on f in $x' = f(t,x)$ may cost uniqueness. Illustrate using an example (hint: look at your previous HW).

   (c) Prove existence of solutions without using the Lipschitz condition.

3. (Gronwall's inequality) Prove Gronwall's inequality: Let $u : [0, \alpha] \to \mathbb{R}$ be continuous and nonnegative. Suppose $C \geq 0$ and $K \geq 0$ are such that $u(t) \leq C + \int_0^t Ku(s)ds$ for all $t \in [0, \alpha]$. Then for all $t \in [0, \alpha]$, $u(t) \leq Ce^{Kt}$.

4. (Continuous dependence on initial conditions) Why is this an important property for a dynamical system to have?